22. The largest sphere contained in the first octant must have a radius equal to the minimum distance from the center $(5, 4, 9)$ to any of the three coordinate planes. The shortest such distance is to the xz-plane, a distance of 4. Thus an equation of the sphere is $(x-5)^2 + (y-4)^2 + (z-9)^2 = 16$.

32. The inequality $x^2 + y^2 + z^2 > 2z \quad \Leftrightarrow \quad x^2 + y^2 + (z-1)^2 > 1$ is equivalent to $\sqrt{x^2 + y^2 + (z-1)^2} > 1$, so the region consists of those points whose distance from the point $(0, 0, 1)$ is greater than 1. This is the set of all points outside the sphere with radius 1 and center $(0, 0, 1)$.

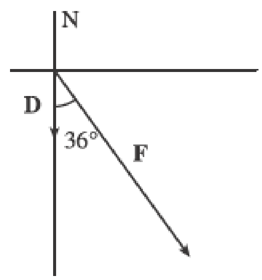
30. Set up the coordinate axes so that north is the positive y-direction, and east is the positive x-direction. The wind is blowing at 50 km/h from the direction N45°W, so that its velocity vector is 50 km/h S45°E, which can be written as $\mathbf{v}_{\text{wind}} = 50(\cos 45°\mathbf{i} - \sin 45°\mathbf{j})$. With respect to the still air, the velocity vector of the plane is 250 km/h N 60°E, or equivalently $\mathbf{v}_{\text{plane}} = 250(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$. The velocity of the plane relative to the ground is

$$\begin{aligned} \mathbf{v} = \mathbf{v}_{\text{wind}} + \mathbf{v}_{\text{plane}} &= (50\cos 45° + 250\cos 30°)\,\mathbf{i} + (-50\sin 45° + 250\sin 30°)\,\mathbf{j} \\ &= \left(25\sqrt{2} + 125\sqrt{3}\right)\mathbf{i} + \left(125 - 25\sqrt{2}\right)\mathbf{j} \approx 251.9\,\mathbf{i} + 89.6\,\mathbf{j} \end{aligned}$$

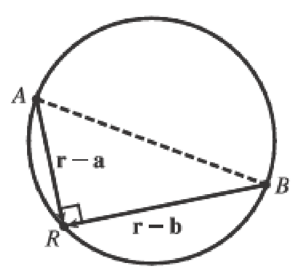
The ground speed is $|\mathbf{v}| \approx \sqrt{(251.9)^2 + (89.6)^2} \approx 267$ km/h. The angle the velocity vector makes with the x-axis is $\theta \approx \tan^{-1}\left(\frac{89.6}{251.9}\right) \approx 20°$. Therefore, the true course of the plane is about N$(90-20)$°E $=$ N 70°E.

42. Let P_1 and P_2 be the points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively. Then $|\mathbf{r} - \mathbf{r}_1| + |\mathbf{r} - \mathbf{r}_2|$ is the sum of the distances from (x, y) to P_1 and P_2. Since this sum is constant, the set of points (x, y) represents an ellipse with foci P_1 and P_2. The condition $k > |\mathbf{r}_1 - \mathbf{r}_2|$ assures us that the ellipse is not degenerate.

48. $W = \mathbf{F} \cdot \mathbf{D} = |\mathbf{F}|\,|\mathbf{D}| \cos\theta = (400)(120)\cos 36° \approx 38{,}833$ ft-lb



50. $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$ implies that the vectors $\mathbf{r} - \mathbf{a}$ and $\mathbf{r} - \mathbf{b}$ are orthogonal. From the diagram (in which A, B and R are the terminal points of the vectors), we see that this implies that R lies on a sphere whose diameter is the line from A to B. The center of this circle is the midpoint of AB, that is, $\frac{1}{2}(\mathbf{a} + \mathbf{b}) = \left\langle \frac{1}{2}(a_1 + b_1), \frac{1}{2}(a_2 + b_2), \frac{1}{2}(a_3 + b_3) \right\rangle$, and its radius is $\frac{1}{2}|\mathbf{a} - \mathbf{b}| = \frac{1}{2}\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$.



Or: Expand the given equation, substitute $\mathbf{r} \cdot \mathbf{r} = x^2 + y^2 + z^2$ and complete the squares.

30. (a) $\overrightarrow{PQ} = \langle -3, 2, -1 \rangle$ and $\overrightarrow{PR} = \langle 1, -1, 1 \rangle$, so a vector orthogonal to the plane through P, Q, and R is

$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle (2)(1) - (-1)(-1), (-1)(1) - (-3)(1), (-3)(-1) - (2)(1) \rangle = \langle 1, 2, 1 \rangle$ (or any scalar mutiple thereof).

(b) The area of the parallelogram determined by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is $\left| \overrightarrow{PQ} \times \overrightarrow{PR} \right| = |\langle 1, 2, 1 \rangle| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$, so the area of triangle PQR is $\frac{1}{2}\sqrt{6}$.