Math 217 Assignment 7

Due Friday November 20

■ Problems from the text (do NOT turn in these problems):

- Section 17.1: 1–16, 33–36.
- Section 17.2: 1–22, 29–34, 39–48.
- Section 17.3: 3–10, 12–24,26–34.

■ Problems to turn in:

1. Sketch the vector field $\mathbf{F}(x-y) = (x-y)\mathbf{i} + x\mathbf{j}$.
2. Sketch the gradient vector field of $f(x,y) = \sqrt{x^2+y^2}$.
3. At time $t = 1$, a particle is located at position $(1,3)$. If it moves in a velocity field

$$\mathbf{F}(x,y) = \langle xy - 2, y^2 - 10 \rangle$$

find its approximate location at time $t = 1.05$.

4. Evaluate

(a) the integral

$$\int_C \sin x\, dx + \cos y\, dy,$$

where C consists of the top half of the circle $x^2 + y^2 = 1$ from $(1,0)$ to $(-1,0)$ and the line segment from $(-1,0)$ and $(-2,3)$.

(b) the integral

$$\int_C x^2\, dx + y^2\, dy + z^2\, dz,$$

where C consists of the line segments from $(0,0,0)$ to $(1,2,-1)$ and from $(1,2,-1)$ to $(3,2,0)$.

5. Find the work done by the force field $\mathbf{F}(x,y) = x \sin y \mathbf{i} + y\mathbf{j}$ on a particle that moves along the parabola $y = x^2$ from $(-1,1)$ to $(2,4)$.
6. The base of a vertical fence is the circle centered at the origin of raidus 10 meters. The height of the fence at position (x,y) is given by the function

$$h(x,y) = 4 + 0.01(x^2 - y^2).$$

Sketch the fence, and find its highest and lowest points. Supposing that a litre of paint covers 100 square meters, determine how much paint would be needed to paint both sides of the fence.

7. Experiments show that a steady current I in a long wire produces a magnetic field $\mathbf{B}$ that is tangent to any circle that lies in the plane perpendicular to the wire and whose center is the axis of the wire. Ampere's law relates the electric current to its magnetic effects and states that

$$\int_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I$$

where I is the net current that passes through any surface bounded by a closed curve C, and μ_0 is a constant called the permeability of free space. By taking C to be a circle with radius r, show that the magnitude $B = |\mathbf{B}|$ of the magnetic field at a distance r from the center of the wire is

$$B = \frac{\mu_0 I}{2\pi r}.$$

8. Verify that the vector field

$$\mathbf{F}(x, y, z) = (2xz + y^2)\mathbf{i} + 2xy\mathbf{j} + (x^2 + 3z^2)\mathbf{k}$$

is conservative. Use this fact to evaluate the lie integral of $\mathbf{F}$ on the curve

$$C : \mathbf{r}(t) = t^2\mathbf{i} + (t + 1)\mathbf{j} + (2t - 1)\mathbf{k},\ 0 \leq t \leq 1.$$

9. Let $\mathbf{F} = \nabla f$ where $f(x, y) = \sin(x - 2y)$. Find curves C_1 and C_2 that are not closed and satisfy the equations

$$(a) \int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 0 \qquad (b) \int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 1.$$

10. Show that the line integral

$$\int_C y\,dx + x\,dy + xyz\,dz$$

is not independent of path.

11. Determine whether the set

$$\{(x, y) : x^2 + y^2 \leq 1 \text{ or } 4 \leq x^2 + y^2 \leq 9\}$$

is (a) open, (b) connected, and (c) simply connected.