12. The solid is $\{(\rho,\theta,\phi) \mid 1 \le \rho \le 2, 0 \le \theta \le \frac{\pi}{2}, 0 \le \phi \le \frac{\pi}{2}\}$ which is the region in the first octant on or between the two spheres $\rho = 1$ and $\rho = 2$.

34. The paraboloid and the half-cone intersect when $x^2 + y^2 = \sqrt{x^2 + y^2}$, that is when $x^2 + y^2 = 1$ or 0. So

$$V = \iint\limits_{x^2+y^2\le 1} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} dz\, dA = \int_0^{2\pi}\int_0^1\int_{r^2}^{r} r\, dz\, dr\, d\theta = \int_0^{2\pi}\int_0^1 (r^2 - r^3)\, dr\, d\theta = \int_0^{2\pi}\left(\tfrac{1}{3} - \tfrac{1}{4}\right) d\theta = \tfrac{1}{12}(2\pi) = \tfrac{\pi}{6}.$$

40. The region of integration is the solid hemisphere $x^2 + y^2 + z^2 \le 4$, $x \ge 0$.

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\, dz\, dx\, dy$$

$$= \int_{-\pi/2}^{\pi/2}\int_0^{\pi}\int_0^2 (\rho\sin\phi\sin\theta)^2\left(\sqrt{\rho^2}\right)\rho^2\sin\phi\, d\rho\, d\phi\, d\theta = \int_{-\pi/2}^{\pi/2}\sin^2\theta\, d\theta \int_0^{\pi}\sin^3\phi\, d\phi \int_0^2 \rho^5\, d\rho$$

$$= \left[\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right]_{-\pi/2}^{\pi/2}\left[-\tfrac{1}{3}(2+\sin^2\phi)\cos\phi\right]_0^{\pi}\left[\tfrac{1}{6}\rho^6\right]_0^2 = \left(\tfrac{\pi}{2}\right)\left(\tfrac{2}{3}+\tfrac{2}{3}\right)\left(\tfrac{32}{3}\right) = \tfrac{64}{9}\pi$$

44. Each lamp has exponential density function

$$f(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{800} e^{-t/800} & \text{if } t \geq 0 \end{cases}$$

If X, Y, and Z are the lifetimes of the individual bulbs, then X, Y, and Z are independent, so the joint density function is the product of the individual density functions:

$$f(x, y, z) = \begin{cases} \frac{1}{800^3} e^{-(x+y+z)/800} & \text{if } x \geq 0, y \geq 0, z \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The probability that all three bulbs fail within a total of 1000 hours is $P(X + Y + Z \leq 1000)$, or equivalently $P((X, Y, Z) \in E)$ where E is the solid region in the first octant bounded by the coordinate planes and the plane $x + y + z = 1000$. The plane $x + y + z = 1000$ meets the xy-plane in the line $x + y = 1000$, so we have

$$\begin{aligned} P(X+Y+Z \leq 1000) &= \iiint_E f(x,y,z)\,dV = \int_0^{1000} \int_0^{1000-x} \int_0^{1000-x-y} \tfrac{1}{800^3} e^{-(x+y+z)/800}\,dz\,dy\,dx \\ &= \tfrac{1}{800^3} \int_0^{1000} \int_0^{1000-x} -800 \Big[e^{-(x+y+z)/800} \Big]_{z=0}^{z=1000-x-y} dy\,dx \\ &= \tfrac{-1}{800^2} \int_0^{1000} \int_0^{1000-x} [e^{-5/4} - e^{-(x+y)/800}]\,dy\,dx \\ &= \tfrac{-1}{800^2} \int_0^{1000} \Big[e^{-5/4} y + 800 e^{-(x+y)/800} \Big]_{y=0}^{y=1000-x} dx \\ &= \tfrac{-1}{800^2} \int_0^{1000} [e^{-5/4}(1800 - x) - 800 e^{-x/800}]\,dx \\ &= \tfrac{-1}{800^2} \Big[-\tfrac{1}{2} e^{-5/4} (1800 - x)^2 + 800^2 e^{-x/800} \Big]_0^{1000} \\ &= \tfrac{-1}{800^2} \Big[-\tfrac{1}{2} e^{-5/4} (800)^2 + 800^2 e^{-5/4} + \tfrac{1}{2} e^{-5/4} (1800)^2 - 800^2 \Big] \\ &= 1 - \tfrac{97}{32} e^{-5/4} \approx 0.1315 \end{aligned}$$

48. $\dfrac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} 2u & 0 & 0 \\ 0 & 2v & 0 \\ 0 & 0 & 2w \end{vmatrix} = 8uvw$, so

$$\begin{aligned} V &= \iiint_E dV = \int_0^1 \int_0^{1-u} \int_0^{1-u-v} 8uvw\,dw\,dv\,du = \int_0^1 \int_0^{1-u} 4uv(1-u-v)^2\,du \\ &= \int_0^1 \int_0^{1-u} \left[4u(1-u)^2 v - 8u(1-u)v^2 + 4uv^3\right] dv\,du \\ &= \int_0^1 \left[2u(1-u)^4 - \tfrac{8}{3}u(1-u)^4 + u(1-u)^4\right] du = \int_0^1 \tfrac{1}{3}u(1-u)^4 du \\ &= \int_0^1 \tfrac{1}{3}\left[(1-u)^4 - (1-u)^5\right] du = \tfrac{1}{3}\left[-\tfrac{1}{5}(1-u)^5 + \tfrac{1}{6}(1-u)^6\right]_0^1 = \tfrac{1}{3}\left(-\tfrac{1}{6} + \tfrac{1}{5}\right) = \tfrac{1}{90} \end{aligned}$$