Homework 1 - Math 440/508, Fall 2011

Due on Monday September 19

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1. Assume a function f is analytic in a domain D and satisfies at least one of the following two conditions:
   - i. $\operatorname{Arg} f(z) = \alpha \ \forall z \in D$, or
   - ii. $v(z) = u(z)^2 \ \forall z \in D$.

   Prove that f is constant on D.

2. Are the following sets connected or disconnected? Give reasons for your answer.
   - i. $F = \{z : \operatorname{Im}(z)/\operatorname{Re}(z) \in \mathbb{Q}\}$
   - ii. $A = \mathbb{C} \setminus \{z : \operatorname{Re}(z), \operatorname{Im}(z) \in \mathbb{Q}\}$

3. Find the entire function $f(z) = u(z) + iv(z)$ such that $f(0) = i$ and $u(z) = 2x^3 y - 2xy^3 + x^2 - y^2$.

4. From Chapter 1, *Complex Analysis - Stein & Shakarchi*: 7, 9, 16(e), 22, 25