Homework 1 - Math 440/508, Fall 2012

Due Friday September 28 at the beginning of lecture.

Instructions: Your homework will be graded both on mathematical correctness and quality of exposition. Please pay attention to the presentation of your solutions.

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1. Show that a holomorphic function cannot have constant modulus without reducing to a constant.

2. Show that $f(z)$ is holomorphic if and only if $\overline{f(\overline{z})}$ is.

3. For which values of z are the following series convergent?

$$\text{(a)} \quad \sum_{n=0}^{\infty} \left(\frac{z}{1+z} \right)^n, \qquad\qquad \text{(b)} \quad \sum_{n=0}^{\infty} \frac{z^n}{1+z^{2n}}.$$

4. Compute

$$\int_\gamma x\, dz$$

where
(a) γ is the directed line segment from 0 to $1+i$.
(b) γ is the circle $|z| = r$, oriented counterclockwise.

5. Suppose that $f(z)$ is analytic on a region that contains a closed curve γ. Show that

$$\int_\gamma \overline{f(z)} f'(z)\, dz$$

is purely imaginary. You may use without proof that f' is continuous.

6. If $P(z)$ is a polynomial and C denotes the circle $|z-a| = R$, what is the value of $\int_C P(z) d\bar{z}$?