

Math 421/510, Spring 2007, Homework Set 3

(Suggested due date: Friday March 2)

Instructions

- Homework will be collected at the end of lecture on Friday.
 - You are encouraged to discuss homework problems among yourselves. Also feel free to ask the instructor for hints and clarifications. However the written solutions that you submit should be entirely your own.
 - Answers should be clear, legible, and in complete English sentences. If you need to use results other than the ones discussed in class, provide self-contained proofs.
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1. During our discussion of the Riesz Representation Theorem we constructed (based on a positive linear functional Λ on $C_c(X)$) a measure μ on X and a collection \mathfrak{M} of subsets of X . Verify the statements regarding μ and \mathfrak{M} that were outlined in steps 2–9 of the lecture on Friday February 16.
2. (Reading exercise) The version of Riesz Representation Theorem stated in class was for a positive linear functional. Read Appendix C in Conway to see how this result can be used to obtain for every linear functional F in $C_c(X)^*$ a measure $\mu \in M(X)$ that represents F . In fact, the map $\mu \mapsto F_\mu$ where

$$F_\mu(f) = \int f d\mu, \quad f \in C_c(X),$$

is an isometric isomorphism of $M(X)$ onto $C_c(X)^*$.