Math 121 Review, Practice Problem Set 1
(Based on Chapter 9)

1. Suppose $a_n > 0$ and $a_{n+1}/a_n \geq n/(n+1)$ for all n. Determine whether $\sum_n a_n$ converges or diverges.
2. If
$$S(x) = \int_0^x \sin(t^2)\, dt, \quad \text{find} \quad \lim_{x \to 0} \frac{x^3 - 3S(x)}{x^7}.$$
3. Find the Maclaurin polynomial of degree 4 of the function $F(x) = \sqrt{1 + \sin x}$.
4. Which function has Maclaurin series
$$1 - \frac{x}{2!} + \frac{x^2}{4!} - \cdots ?$$
5. What is the Fourier series of the 2π periodic function $h(t) = \cos^2 t$?
6. Write down the Fourier series of the 2π-periodic function
$$f(t) = \pi - |t|, \quad -\pi \leq t < \pi,$$
and use it to evaluate the series
$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}.$$