10. $x = \cos t + \sin t$, $y = \cos t - \sin t$, $(0 \leq t \leq 2\pi)$
The circle $x^2 + y^2 = 2$, traversed clockwise, starting and ending at $(1, 1)$.

17. $x = e^t - t$, $y = 4e^{t/2}$, $(0 \le t \le 2)$. Length is

$$\begin{aligned} L &= \int_0^2 \sqrt{(e^t - 1)^2 + 4e^t}\, dt \\ &= \int_0^2 \sqrt{(e^t + 1)^2}\, dt = \int_0^2 (e^t + 1)\, dt \\ &= (e^t + t)\Big|_0^2 = e^2 + 1 \text{ units.} \end{aligned}$$

25. Area of a large loop:

$$\begin{aligned} A &= 2 \times \frac{1}{2} \int_0^{\pi/3} (1 + 2\cos(2\theta))^2 \, d\theta \\ &= \int_0^{\pi/3} [1 + 4\cos(2\theta) + 2(1 + \cos(4\theta))] \, d\theta \\ &= \left(3\theta + 2\sin(2\theta) + \frac{1}{2}\sin(4\theta)\right)\Big|_0^{\pi/3} \\ &= \pi + \frac{3\sqrt{3}}{4} \text{ sq. units.} \end{aligned}$$

5.

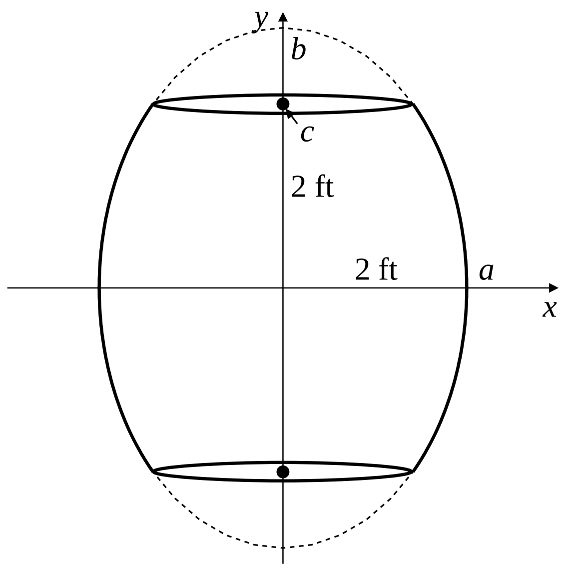

Fig. C-5

Let the ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, with $a = 2$ and foci at $(0, \pm 2)$ so that $c = 2$ and $b^2 = a^2 + c^2 = 8$. The volume of the barrel is

$$V = 2\int_0^2 \pi x^2\, dy = 2\pi \int_0^2 4\left(1 - \frac{y^2}{8}\right) dy$$

$$= 8\pi \left(y - \frac{y^3}{24}\right)\Big|_0^2 = \frac{40\pi}{3}\ \text{ft}^3.$$

16. If $r^2 = \cos 2\theta$, then

$$2r\frac{dr}{d\theta} = -2\sin 2\theta \Rightarrow \frac{dr}{d\theta} = -\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$$

and

$$ds = \sqrt{\cos 2\theta + \frac{\sin^2 2\theta}{\cos 2\theta}}\, d\theta = \frac{d\theta}{\sqrt{\cos 2\theta}}.$$

a) Area of the surface generated by rotation about the x- axis is

$$\begin{aligned} S_x &= 2\pi \int_0^{\pi/4} r \sin\theta \, ds \\ &= 2\pi \int_0^{\pi/4} \sqrt{\cos 2\theta}\, \sin\theta \frac{d\theta}{\sqrt{\cos 2\theta}} \\ &= -2\pi \cos\theta \Big|_0^{\pi/4} = (2-\sqrt{2})\pi \text{ sq. units.} \end{aligned}$$

b) Area of the surface generated by rotation about the y- axis is

$$\begin{aligned} S_y &= 2\pi \int_{-\pi/4}^{\pi/4} r \cos\theta \, ds \\ &= 4\pi \int_0^{\pi/4} \sqrt{\cos 2\theta}\, \cos\theta \frac{d\theta}{\sqrt{\cos 2\theta}} \\ &= 4\pi \sin\theta \Big|_0^{\pi/4} = 2\sqrt{2}\pi \text{ sq. units.} \end{aligned}$$

3. A strip along the slant wall of the dam between depths h and $h + dh$ has area

$$dA = \frac{200\,dh}{\cos\theta} = 200 \times \frac{26}{24}\,dh.$$

The force on this strip is

$$dF = 9,800\,h\,dA \approx 2.12 \times 10^6\,h\,dh \text{ N}.$$

Thus the total force on the dam is

$$F = 2.12 \times 10^6 \int_0^{24} h\,dh \approx 6.12 \times 10^8 \text{ N}.$$

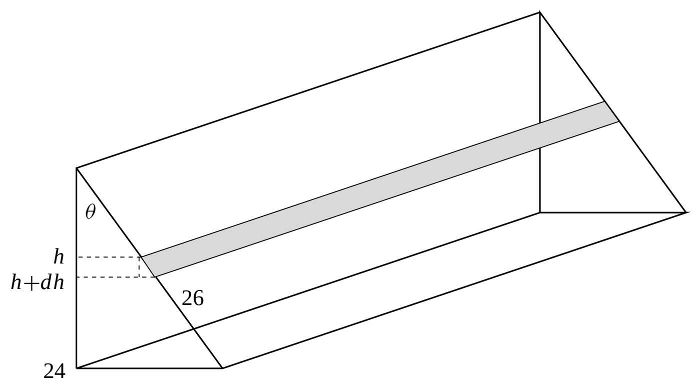

Fig. 6-3