17. $$\begin{aligned} K(x) &= \int_1^{1+x} \frac{\ln t}{t-1}\,dt \qquad \text{let } u = t-1 \\ &= \int_0^x \frac{\ln(1+u)}{u}\,du \\ &= \int_0^x \left[1 - \frac{u}{2} + \frac{u^2}{3} - \frac{u^3}{4} + \cdots\right] du \\ &= x - \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + \cdots \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{(n+1)^2} \quad (-1 \le x \le 1) \end{aligned}$$

19.

$$
\begin{aligned}
M(x) &= \int_0^x \frac{\tan^{-1}(t^2)}{t^2}\,dt \\
&= \int_0^x \left[1 - \frac{t^4}{3} + \frac{t^8}{5} - \frac{t^{12}}{7} + \cdots\right] dt \\
&= x - \frac{x^5}{3\times 5} + \frac{x^9}{5\times 9} - \frac{x^{13}}{7\times 13} + \cdots \\
&= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(2n+1)(4n+1)} \qquad (-1 \le x \le 1)
\end{aligned}
$$

24. We have

$$\lim_{x\to 0}\frac{(e^x-1-x)^2}{x^2-\ln(1+x^2)}=\lim_{x\to 0}\frac{\left(\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots\right)^2}{\frac{x^4}{2}-\frac{x^6}{3}+\frac{x^8}{4}-\cdots}$$

$$=\lim_{x\to 0}\frac{\frac{x^4}{4}\left(1+\frac{x}{3}+\frac{x^2}{12}+\cdots\right)^2}{\frac{x^4}{2}-\frac{x^6}{3}+\frac{x^8}{4}-\cdots}=\frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}=\frac{1}{2}.$$

26. We have

$$\lim_{x\to 0}\frac{\sin(\sin x)-x}{x[\cos(\sin x)-1]}$$

$$=\lim_{x\to 0}\frac{\left(\sin x-\frac{1}{3!}\sin^3 x+\frac{1}{5!}\sin^5 x-\cdots\right)-x}{x\left[1-\frac{1}{2!}\sin^2 x+\frac{1}{4!}\sin^4 x-\cdots-1\right]}$$

$$=\lim_{x\to 0}\frac{\left(x-\frac{x^3}{3!}+\cdots\right)-\frac{1}{3!}\left(x-\frac{x^3}{3!}+\cdots\right)^3+\frac{1}{5!}\left(x-\cdots\right)^5-\cdots-x}{x\left[-\frac{1}{2!}\left(x-\frac{x^3}{3!}+\cdots\right)^2+\frac{1}{4!}\left(x-\cdots\right)^4-\cdots\right]}$$

$$=\lim_{x\to 0}\frac{-\frac{2}{3!}x^3+\text{higher degree terms}}{-\frac{1}{2!}x^3+\text{higher degree terms}}=\frac{\frac{2}{3!}}{\frac{1}{2!}}=\frac{2}{3}.$$

8. $f(t) = \begin{cases} t & \text{if } 0 \le t < 1 \\ 1 & \text{if } 1 \le t < 2, \\ 3-t & \text{if } 2 \le t < 3 \end{cases}$ $\quad f$ has period 3.

f is even, so its Fourier sine coefficients are all zero. Its cosine coefficients are

$$\begin{aligned}
\frac{a_0}{2} &= \frac{1}{2}\cdot\frac{2}{3}\int_0^3 f(t)\,dt = \frac{2}{3}(2) = \frac{2}{3} \\
a_n &= \frac{2}{3}\int_0^3 f(t)\,\cos\frac{2n\pi t}{3}\,dt \\
&= \frac{2}{3}\left[\int_0^1 t\cos\frac{2n\pi t}{3}\,dt + \int_1^2 \cos\frac{2n\pi t}{3}\,dt \right. \\
&\qquad \left. + \int_2^3 (3-t)\cos\frac{2n\pi t}{3}\,dt\right] \\
&= \frac{3}{2n^2\pi^2}\left[\cos\frac{2n\pi}{3} - 1 - \cos(2n\pi) + \cos\frac{4n\pi}{3}\right].
\end{aligned}$$

The latter expression was obtained using Maple to evaluate the integrals. If $n = 3k$, where k is an integer, then $a_n = 0$. For other integers n we have $a_n = -9/(2\pi^2 n^2)$. Thus the Fourier series of f is

$$\frac{2}{3} - \frac{9}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\frac{2n\pi t}{3} + \frac{1}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos(2n\pi t).$$

9. i) $\binom{n}{0} = \frac{n!}{0!n!} = 1, \quad \binom{n}{n} = \frac{n!}{n!0!} = 1.$

ii) If $0 \le k \le n$, then

$$\begin{aligned}\binom{n}{k-1} + \binom{n}{k} &= \frac{n!}{(k-1)!(n-k+1)!} + \frac{n!}{k!(n-k)!} \\ &= \frac{n!}{k!(n-k+1)!}\big(k + (n-k+1)\big) \\ &= \frac{(n+1)!}{k!(n+1-k)!} = \binom{n+1}{k}.\end{aligned}$$