Math 121 Practice Problem Set 2 for the second midterm
(Based on Chapter 7)

1. Find a family of curves that intersect every ellipse of the form $3x^2 + 4y^2 = C$ at right angles.
2. A thin plate in the shape of a circular disk has radius 3 ft. A circular hole of radius 1 ft is cut out of the disk, centred 1 ft from the center of the disk. Find the centroid of the remaining part of the disk.
3. Solve the initial-value problem
$$y' + (\cos x)y = 2xe^{-\sin x}, \qquad y(\pi) = 0.$$
4. Consider a random variable whose density function is given by
$$f_{\mu,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty.$$
Find the standard deviation of the random variable.
5. Find the work that must be done to pump all the water in a full hemispherical bowl of radius a meters to a height h meters above the top of the bowl.
6. A *horn* is a surface of revolution obtained by rotating a positive decreasing curve that decays to zero at infinity about the x-axis. Find a horn that can be filled in with a finite amount of paint, but which cannot be painted with a finite amount of paint!