Math 121 Practice Problem Set 2
(Based on Chapters 6 and 7)

1. Determine the type of the following improper integral. Then find out whether it converges or diverges.

$$\int_0^\infty \frac{x^2}{x^5+1}\,dx.$$

2. Evaluate the integrals

$$(a)\ \int \frac{d\theta}{\cos\theta(1+\sin\theta)} \quad \text{and } (b) \quad \int_1^e \sin(\ln x)\,dx.$$

(Answer: (a) $\frac{1}{4}\ln\left|\frac{1+\sin\theta}{1-\sin\theta}\right| - \frac{1}{2(1+\sin\theta)} + C$, (b) $\frac{1}{2}(e\sin(1) - e\cos(1) + 1)$)

3. Consider a square plate of edge length a, whose density at a point P is equal to kr g/cm^2, where r is the distance in centimetres from P to one of the diagonals of the square. Find the mass and center of mass of the plate.

(Answer: $k\frac{a^3}{3\sqrt{2}}$g, centre of the square.)