# Math 105 Assignment 10 Solutions

1. A star player in the NBA is offered a 6-year contract by a team and two choices for compensation. In the first, he is offered a lump sum of $40,000,000, paid at the beginning of his contract. In the second, he is offered an initial payment of $6,000,000 and a 6-year continuous income stream at the rate of $7,500,000 per year deposited into a savings account paying 8% annual interest, compounded continuously. Assuming that the player can also invest his money with the same interest of 8%, determine which plan is better for the player, and by how much.

   **Solution:** We calculate the present value of each option. Option one has a present value of $40,000,000, and option two has a present value of $\$6,000,000+$ PV, where PV is the present value of being paid $7,500,000 per year deposited into a savings account paying 8% continuously compounded annual interest. We now calculate this present value.

   From the formula for present value of a continuous income stream, we have PV $= \int_0^7 7,500,000e^{-0.08t}dt \approx 35,739,057$. Thus the second option pays $41,739,057 in present value, so it is the better option.

2. A random variable has only three possible values: 1, 2 and 4. The expected value (mean) is 3 and the variance is $\frac{3}{2}$. Find the probability distribution of $X$.

   **Solution:** We let $p_1$ be the probability of 1, $p_2$ the probability of 2, and $p_3$ the probability of 4.

   The expected value of the probability distribution is $1p_1 + 2p_2 + 4p_3 = 3$ and the variance is $(1-3)^2p_1 + (2-3)^2p_2 + (4-3)^2p_3 = 4p_1 + p_2 + p_3 = \frac{3}{2}$. Together with the fact that $p_1 + p_2 + p_3 = 1$ (probability distribution), this gives us a system of three equations we can solve to find the probabilities $p_1, p_2, p_3$.

   Subtract twice the second equation from the first to get $-7p_1 + 2p_3 = 0$, which gives $p_3 = \frac{7}{2}p_1$. Substituting this into the first equation, we have $15p_1 + 2p_2 = 3$, from which we get $p_2 = \frac{3-15p_1}{2}$.

   Plugging this information into $p_1+p_2+p_3 = 1$, we get $p_1 + \frac{3-15p_1}{2} + 7/2p_1 = -3p_1 + \frac{3}{2} = 1$, which gives $p_1 = \frac{1}{6}$, so $p_2 = \frac{1}{4}$ and $p_3 = \frac{7}{12}$.

3. Assume that the daily demand for a certain product in thousands of units has probability density function

$$f(x) = \frac{1}{18}(9 - x^2), \qquad 0 \le x \le 3.$$

(a) Find the probability that the demand is at least 1000 units.
(b) Find the probability that the demand is at most 2000 units.
(c) Find the probability that the demand is between 1000 and 2000 units.

**Solution:**

(a) This is the integral $\int_1^3 f(x)dx = \left[\frac{1}{18}(9x - \frac{x^3}{3})\right]_1^3 = \frac{14}{27}$

(b) This is the integral $\int_0^2 f(x)dx = \frac{23}{27}$

(c) This is the integral $\int_1^2 f(x)dx = \frac{10}{27}$