

2. (a) Use integration by parts to calculate the definite integral (5 pts)

$$I = \int_{-1}^1 x^2 \sin x \, dx$$

$$\int uv' = uv - \int u'v$$

$$u = x^2, \quad v' = \sin(x) \Rightarrow v = -\cos(x), \quad u' = 2x$$

$$I = \left. -x^2 \cos(x) \right|_{-1}^1 - \int_{-1}^1 2x (-\cos(x)) \, dx$$

$$= \underbrace{[-1 \cos(1)] - [-1 \cos(-1)]}_{=0 \text{ since } \cos(a) = \cos(-a)} + \int_{-1}^1 \underbrace{2x}_u \underbrace{\cos(x)}_{v'} \, dx$$

$$u = 2x, \quad v' = \cos(x) \Rightarrow u' = 2, \quad v = \sin(x)$$

$$I = \left. 2x \sin(x) \right|_{-1}^1 - \int_{-1}^1 2 \sin(x) \, dx$$

$$= \underbrace{2 \sin(1) - (-2 \sin(-1))}_{=0 \text{ since } \sin(-1) = -\sin(1)} - \int_{-1}^1 2 \sin(x) \, dx$$

$$= 2 \cos(x) \Big|_{-1}^1 = 2 \cos(1) - 2 \cos(-1) = 0, \text{ since } \cos(1) = \cos(-1).$$