

(b) Is there any other simpler way to calculate the integral? (Hint: You do not need to find the antiderivative of the function.) (1 pt)

$f(x) = x^2 \sin(x)$ is an odd function, i.e.,
 $f(x) = -f(-x)$. So, its integral over any symmetric interval $[-a, a]$ is zero.

3. (a) What is the area between the graph of $f(x) = \frac{e^x}{\sqrt{1-e^{2x}}}$ and the x -axis on the interval $[\ln \frac{1}{2}, 0]$? (4 pts)

$$I = \int_{\ln(\frac{1}{2})}^0 \frac{e^x}{\sqrt{1-e^{2x}}} dx.$$

Let $u = e^x$. Then $du = e^x dx$.

$$x=0 \Rightarrow u = e^0 = 1$$

$$x = \ln(\frac{1}{2}) \Rightarrow u = e^{\ln(\frac{1}{2})} = \frac{1}{2}$$

$$\begin{aligned} I &= \int_{\frac{1}{2}}^1 \frac{du}{\sqrt{1-u^2}} = \arcsin(u) \Big|_{\frac{1}{2}}^1 \\ &= \arcsin(1) - \arcsin\left(\frac{1}{2}\right) \\ &= \frac{\pi}{2} - \frac{\pi}{6} \\ &= \frac{\pi}{3} \end{aligned}$$