Math 105 Assignment 6 Solutions

1. (5 points) Find an equation for a plane P which is parallel to $3x + y + z = 9$, and contains the point $(1, 5, 6)$.

Solution: The plane $3x + y + z = 9$ has normal vector $\mathbf{n} = (3, 1, 1)$, so if P is parellel to this plane, then $\mathbf{n}$ is a normal vector for P.

The equation for a plane P passing through $(1, 5, 6)$ with normal vector $(3, 1, 1)$ is $3(x - 1) + (y - 5) + (z - 6) = 0$, or $3x + y + z = 14$.

2.a (3 points) Let $f(x, y) = 100x^{\frac{1}{3}}y^{\frac{2}{3}}$, and let C be a positive constant. Express the level curve $f(x, y) = C$ as the graph of a function $y = g(x)$.

Solution: We take the equation $100x^{\frac{1}{3}}y^{\frac{2}{3}} = C$ and isolate y.

First we rearrange to get $y^{\frac{2}{3}} = \frac{C}{100}x^{\frac{-1}{3}}$. Cubing each side gives $y^2 = \frac{1}{100^3}C^3x^{-1}$.

Now taking square roots, we get two branches of y as a function of x for the positive and negative square roots, $y = \frac{1}{1000}C^{\frac{3}{2}}x^{\frac{-1}{2}}$ and $y = -\frac{1}{1000}C^{\frac{3}{2}}x^{\frac{-1}{2}}$.

2.b (2 points) Describe the level curve $f(x, y) = C$ when $C = 0$, and when $C < 0$.

Solution: $f(x, y) = 100x^{\frac{1}{3}}y^{\frac{2}{3}} = 0$ exactly when one of x or y is zero. The level curve is then the two lines $x = 0$ and $y = 0$ (a degenerate hyperbola).

When $f(x, y) = 100x^{\frac{1}{3}}y^{\frac{2}{3}} = C < 0$, the level curve is the graph of the function $x = h(y)$, where $h(y) = \frac{1}{1000000}C^3y^{-2}$ (note that we can't take the square root of C like we did in part a).

3. (5 points) Determine whether or not the following limit exists:

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y}{x^2 - y}.$$

Solution: The limit does not exist. Letting $f(x, y) = \frac{x^2+y}{x^2-y}$, we show that $f(x, y)$ approaches two different values as (x, y) approaches $(0, 0)$ along two different paths.

Consider the limits approaching from the lines $y = 0$ and $x = 0$ from the positive x direction and positive y direction, respectively.

For the line $y = 0$, we have

$$\lim_{(x,y)\to(0,0)} f(x, y) = \lim_{x\to 0^+} \frac{x^2 - 0}{x^2 + 0} = 1$$

For the line $x = 0$, we have

$$\lim_{(x,y)\to(0,0)} f(x, y) = \lim_{y\to 0^+} \frac{0^2 - y}{0^2 + y} = -1$$

Since these are not the same, the limit can't exist.