Lesson plan: Constrained optimization

Statement of the problem

Given a function $f(x, y)$ (called the **objective function**), we need to find the smallest and largest values of f subject to the **constraint**

$$g(x, y) = 0.$$

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The technique used to solve the problem above is called **the method of Lagrange multipliers**.

Method of Lagrange multipliers

Steps in the solution

- Find the values of x, y and λ that satisfy the equations

$$\nabla f(x, y) = \lambda \nabla g(x, y) \quad \text{and} \quad g(x, y) = 0.$$

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- Among the solutions (x, y) that you found in the previous step, select the ones which gives largest and smallest values of the function f. These are the maximum and minimum values of the objective function subject to the constraint.

An example

Find the maximum value of $36 - x^2 - y^2$ subject to the constraint $x + 7y - 25 = 0$.

A. 36

B. 32

C. $47/2$

D. 18

E. 20

Production optimization

Suppose that x units of labor and y units of capital can produce $f(x, y) = 60x^{\frac{3}{4}}y^{\frac{1}{4}}$ units of a certain product. Also suppose that each unit of labor costs \$100, whereas each unit of capital costs \$200. Assume that \$30,000 is available to spend on production. How many units of labor and how many units of capital should be utilized to maximize production?

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A. 37.5 units of labor and 225 units of capital

B. 50 units of capital and 60 units of labor

C. 225 units of labor and 37.5 units of capital

D. 60 units of capital and 50 units of labor