Discrete and continuous random variables

Malabika Pramanik

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Review : Discrete Random variable

The probability distribution of a discrete random variable is given by the table

value of X	probability
x_1	p_1
x_2	p_2
$\vdots$	$\vdots$
x_n	p_n
Total	1

which is interpreted as follows:

$$p_k = \Pr(X = x_k) = \text{probability that } X \text{ takes the value } x_k.$$

Continuous random variable

For a continuous random variable X, the probability distribution is represented by means of a function f, satisfying

$$f(x) \geq 0 \text{ for all } x, \qquad \int_{-\infty}^{\infty} f(x)\,dx = 1. \tag{1}$$

Any function f satisfying (1) is called a *probability density function*. The relation between f and X is as follows:

$$\begin{aligned} \text{Prob that } X \text{ takes values in } [a,b] &= \Pr(a \leq X \leq b) \\ &= \int_a^b f(x)\,dx. \end{aligned}$$

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For a continuous random variable with density,

$$\Pr(X = c) = 0 \text{ for any } c!$$

Example

Which of the following functions can be probability density functions?

A. $f(x) = e^{-x}$, $-\infty < x < \infty$.
B. $f(x) = \sin x$, $0 \leq x \leq 5\pi/2$, 0 otherwise.
C. $f(x) = 2x/3$, $-1 \leq x \leq 2$, 0 otherwise.
D. $f(x) = e^{-x}$, $0 < x < \infty$, 0 otherwise.
E. $f(x) = \sin x$, $0 \leq x \leq \pi$, 0 otherwise.

Cumulative distribution function

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The *cumulative distribution function* of X is the function $F(x)$ defined by

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2. $F'(x) = f(x)$. (Fundamental theorem of calculus)
3. F is a non-decreasing function of x.
4. If the random variable X always takes values in the interval $[A, B]$, then $F(A) = 0$ and $F(B) = 1$.

Example

If X is a continuous random variable with density

$$f(x) = 2(x+1)^{-3}, \qquad x \geq 0$$

then the cumulative distribution function for f is

A. $1/(x+1)^2$, $x \geq 0$
B. $(x^2+2x)/(x+1)^2$, $x \geq 0$
C. $-1/(x+1)^2$, $x \geq 0$
D. $2-1/(x+1)^2$, $x \geq 0$
E. $1-e^{-x}$, $x \geq 0$.

Example (ctd)

Use the cumulative distribution function from the preceding example to calculate $\Pr(X \geq 3)$.

A. $1/16$
B. $3/16$
C. 0
D. $2/3$
E. $15/16$