Relative maxima and minina

Definition

Given a function $f(x, y)$ of two variables,

- We say that f has a **local maximum** at the point (a, b) if

  $$f(x, y) \leq f(a, b)$$

  for all (x, y) close enough to (a, b).

- We say that f has a **local minimum** at the point (a, b) if

  $$f(x, y) \geq f(a, b)$$

  for all (x, y) close enough to (a, b).

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Today's goal: Given a function f, identify its local maxima and minima.

The first derivative test

Description of the test

The first step in finding local max or min of a function $f(x, y)$ is to find points (a, b) that satisfy the two equations

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0.$$

Any such point (a, b) is called a **critical point of** f.

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Note: Any local max or min of f has to be a critical point, but every critical point need not be a local max or min.

Finding critical points : an example

Find all critical points of the following function

$$f(x, y) = \frac{1}{x} + \frac{1}{y} + xy.$$

A. $(0,0),(1,1)$

B. $(1,1)$

C. $(0,0),(1,1),(1,-1),(-1,1),(-1,-1)$

D. There is no critical point

E. $(0,0)$

The previous example (ctd)

Is the critical point $(1, 1)$ a local max, a local min or neither?

The second derivative test

If (a, b) is a critical point of f, calculate $D(a, b)$, where

$$D = f_{xx} f_{yy} - f_{xy}^2.$$

1. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a local maximum value at (a, b).
2. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a local minimum value at (a, b).
3. If $D(a, b) < 0$, then f has a saddle point at (a, b).
4. If $D(a, b) = 0$, then the test is inconclusive.

Classifying critical points : an example

In the example

$$f(x,y) = \frac{1}{x} + \frac{1}{y} + xy$$

determine whether the critical point $(1,1)$ is

A. a local minimum

B. a local maximum

C. a saddle point

D. neither of the above

An application

A company manufactures two products A and B that sell for \$10 and \$9 per unit respectively. The cost of producing x units of A and y units of B is

$$400 + 2x + 3y + 0.01(3x^2 + xy + 3y^2).$$

Find the values of x and y that maximize the company's profits.

A. $(100, 80)$

B. $(120, 90)$

C. $(120, 80)$

D. $(80, 120)$