

Applications of integration II : Future and present value of a continuous income stream

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Future value of a continuous income stream

Integral representation of future value

The future value of a continuous income stream flowing at the rate of $S(t)$ dollars per year for T years, earning interest at an annual rate r , compounded continuously is given by

$$FV = \int_0^T S(t)e^{r(T-t)} dt.$$

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Derive the integral formula above.

Present value of a continuous income stream

Present value

The present value of a continuous income stream flowing at the rate of $S(t)$ dollars per year for T years, earning interest at an annual rate r , compounded continuously, is given by

$$PV = \int_0^T S(t)e^{-rt} dt.$$

Example 1 : Basic skills

Find the future value after 10 years of a continuous income stream of \$1200 per year deposited in an account paying 6% annual interest, compounded continuously.

- A. 20,000
- B. $20,000(e^{0.6} - 1)$
- C. $1200e^{0.6}$
- D. $20,000e^{0.6}$

Example 2: Basic skills

Find the present value of a continuous income stream of \$500 per month deposited for 20 years into an account paying 4% annual interest, compounded continuously.

- A. 120,000
- B. $120,000e^{-0.8}$
- C. $12500(1 - e^{-0.8})$
- D. $150,000(1 - e^{-0.8})$

Example 3

Assume that a company manufactures a certain type of machine and it estimates that each machine will generate a continuous income stream whose rate in the t^{th} year of operation will be $15 - 2t$ million dollars per year. Assuming that the lifetime of a machine is about 7 years and that money can be invested at the annual rate of 8%, compounded continuously, which of the integrals below represent the fair market price for each machine?

A. $\int_0^7 (15 - 2t)e^{0.08t} dt$

B. $\int_0^7 (15 - 2t)e^{0.08(7-t)} dt$

C. $\int_0^7 (15 - 2t)e^{-0.08t} dt$

D. $\int_0^7 (15 - 2t)e^{0.08(t-7)} dt$

How would you compute the correct integral above?