Math 105 Week 11

March 21, 2011

1. Lesson Plan

This week we will cover the following concepts:

- constrained optimization using Lagrange multipliers, and
- an application of integration: consumer and producer surplus.

The first of these items is contained in section 12.9 of the textbook. For the second item, see the notes and supplementary problems posted on the course website.

- 1.1. Section 12.9. An extremely important optimization problem with many real-life applications is one where, instead of finding local maxima or minima, one needs to identify largest or smallest values of a function subject to a certain number of rules or constraints. We will spend two lecture hours on developing the method of Lagrange multipliers that addresses such problems. Apart from learning the basic skills involved in this process, setting up of a constrained optimization problem from a word problem should also be emphasized. We will optimize functions of two variables only, subject to a single constraint. As before, functions of three variables will be omitted.
- 1.2. Application of integration: Consumer and producer surplus. The remainder of the course will be devoted to exploring several economic and financial situations where the idea of integration plays a key role. The first application that we will consider this week is the quantitative notion of surplus. It is important that you understand the economic significance of these quantities.

As you will see in the online notes, there is an argument that gives an initial approximation of the surplus quantities in terms of Riemann sums. These are then converted by a limiting process to certain integrals that become the formulae of consumer and producer surplus. The arguments leading up the formulae are more important than the actual formulae themselves, being ubiquitous in most real-life problems involving integration. Please spend some time understanding this line of reasoning.

Only the material in the online notes is covered in the course syllabus, you do not need any other book or external resources for this topic.

2. Learning objectives

By the end of the week and after going through the practice problems,

- 1. you should be able to identify and set up a constrained optimization problem and solve it using the method of Lagrange multipliers,
- 2. you should be familiar with the terms supply curve, demand curve, equilibrium price, equilibrium quantity, market equilibrium point, consumer surplus, producer surplus,
- 3. given supply and demand curves for a commodity, you should be able to calculate the equilibrium point, and compute the consumer surplus and producer surplus,
- 4. you should be able to identify the consumer and producer surplus as certain areas under the supply and demand curves.