The University of British Columbia

Midterm 1 - February 2, 2012

Mathematics 105, 2011W T2

Sections 208, 209

Closed book examination Time: 50 minutes

Last Name ____________ **First** ________ **SID** ____________

Instructor names: Djun Kim, Erin Moulding

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form. Simplification of the final answer is worth at most one point.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

Q	Points	Max
1		50
2		20
3		20
4		10
5 (extra credit)		10
Total		100

1. (a) Let

$$f(x, y) = y^2 + y \ln x.$$

Use the partial derivatives of f at the point $(1, 2)$ to estimate the value of $f(1.1, 1.9)$.

(8 points)

(b) Find a unit vector parallel to $\langle -2, 1, 2 \rangle$.

(8 points)

(c) Find an equation for the plane passing through the point $P(1, 2, 3)$ that is orthogonal to the vector $\langle 4, 0, -1\rangle$.

(8 points)

(d) Compute the right Riemann sum for $f(x) = 5 - x$ in the interval $[-2, 4]$ with $n = 3$ equal subintervals.

(8 points)

(e) Assume that $f(x, y)$ has continuous partial derivatives of all orders. If

$$f_y(x, y) = x^3 + 2x^2y,$$

compute f_{xyx}. State in detail any result that you use.

(8 points)

(f) You are given a function

$$f(x, y) = x^2 - 2x + y^2.$$

Compute the maximum and minimum value of f on the *boundary of the region* R, where

$$R = \left\{ x^2 + y^2 \leq 4,\ x \geq 0 \right\}.$$

(10 points)

2. Find *all* critical points of the function

$$f(x, y) = xy - \frac{x^2}{2} - \frac{y^3}{3} + 5.$$

Classify each point as a local maximum, local minimum, or saddle point.

(10 + 10 = 20 points)

3. A firm produces

$$P(x, y) = x^{\frac{2}{3}} y^{\frac{1}{3}}$$

units of goods per week, utilizing x units of labour and y units of capital. If labour costs \$27 per unit and capital costs \$0.5 per unit, use the method of Lagrange multipliers to find the most cost-efficient division of labour and capital that the firm can adopt if its goal is to produce 6 units of goods per week.

Clearly state the objective function and the constraint. There is no need to justify that the solution you obtained is the absolute max or min. **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**

(20 points)

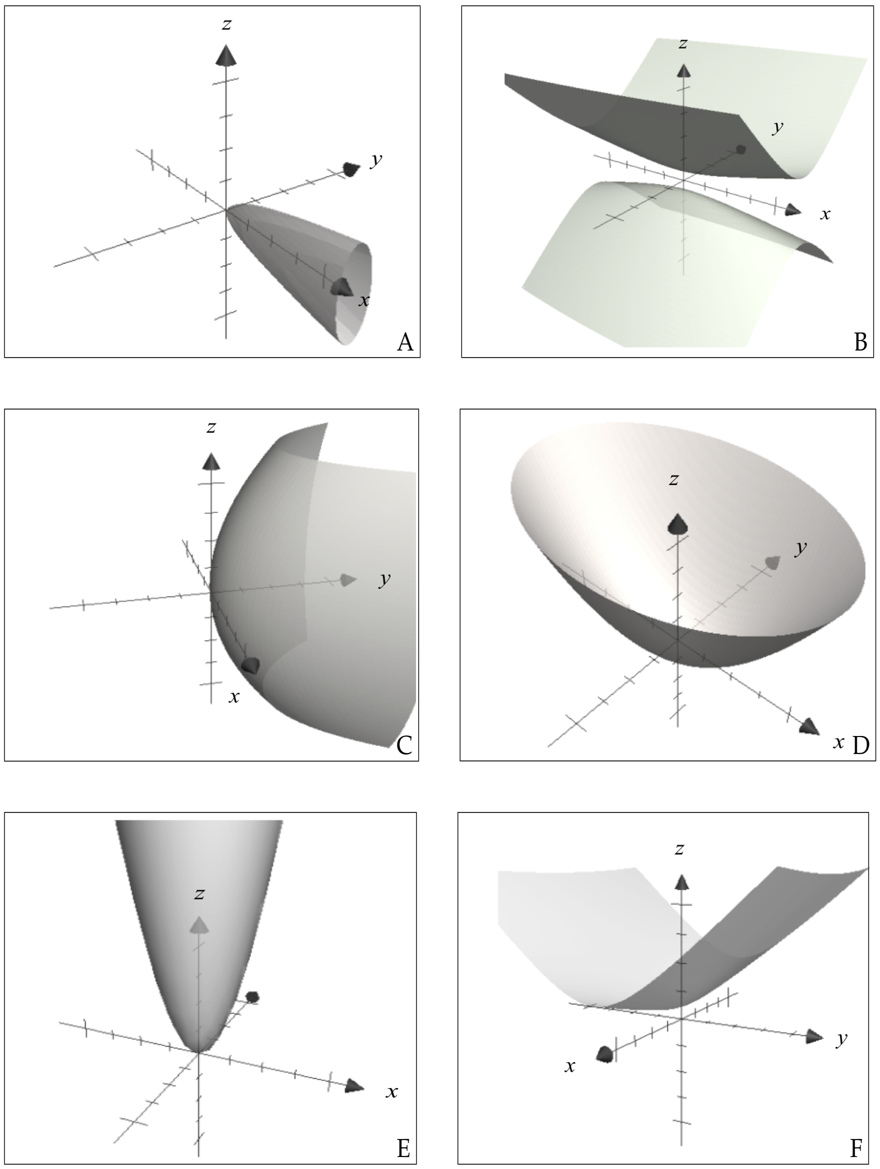
4. Consider the surface S given by
$$z^2 = x - 9y^2.$$

   (a) Sketch the traces of S in the $x = 4$ and $z = 0$ planes, labeling your axes carefully.

(3 + 3 = 6 points)

(b) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?

(4 points)



5. (Extra credit) Find a suitable function $f(x)$ and interval $[a, b]$ so that

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{n} \cos\left(\frac{k}{n}\right).$$

(10 points)

Formula Sheet

You may refer to these formulae if necessary.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$