

The University of British Columbia

Midterm 1 - February 2, 2012

Mathematics 105, 2011W T2

Sections 208, 209

Closed book examination

Time: 50 minutes

Last Name _____ First _____ SID _____

Instructor names: Djun Kim, Erin Moulding

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in “calculator-ready” form. Simplification of the final answer is worth at most one point.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCCard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
 - (b) Speaking or communicating with other candidates.
 - (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Q	Points	Max
1		50
2		20
3		20
4		10
5 (extra credit)		10
Total		100

1. (a) Let

$$f(x, y) = y^2 + y \ln x.$$

Use the partial derivatives of f at the point $(1, 2)$ to estimate the value of $f(1.1, 1.9)$.

(8 points)

(b) Find a unit vector parallel to $\langle -2, 1, 2 \rangle$.

(8 points)

- (c) Find an equation for the plane passing through the point $P(1, 2, 3)$ that is orthogonal to the vector $\langle 4, 0, -1 \rangle$.

(8 points)

- (d) Compute the right Riemann sum for $f(x) = 5 - x$ in the interval $[-2, 4]$ with $n = 3$ equal subintervals.

(8 points)

(e) Assume that $f(x, y)$ has continuous partial derivatives of all orders. If

$$f_y(x, y) = x^3 + 2x^2y,$$

compute f_{xyx} . State in detail any result that you use.

(8 points)

(f) You are given a function

$$f(x, y) = x^2 - 2x + y^2.$$

Compute the maximum and minimum value of f on the *boundary of the region* R , where

$$R = \{x^2 + y^2 \leq 4, x \geq 0\}.$$

(10 points)

2. Find *all* critical points of the function

$$f(x, y) = xy - \frac{x^2}{2} - \frac{y^3}{3} + 5.$$

Classify each point as a local maximum, local minimum, or saddle point.

(10 + 10 = 20 points)

3. A firm produces

$$P(x, y) = x^{\frac{2}{3}}y^{\frac{1}{3}}$$

units of goods per week, utilizing x units of labour and y units of capital. If labour costs \$27 per unit and capital costs \$0.5 per unit, use the method of Lagrange multipliers to find the most cost-efficient division of labour and capital that the firm can adopt if its goal is to produce 6 units of goods per week.

Clearly state the objective function and the constraint. There is no need to justify that the solution you obtained is the absolute max or min. **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**

(20 points)

4. Consider the surface S given by

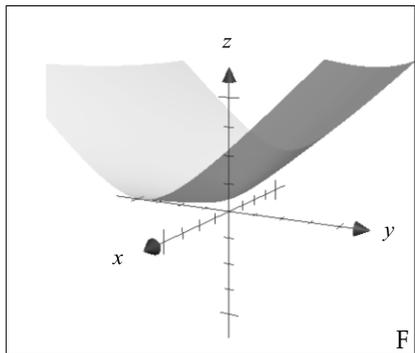
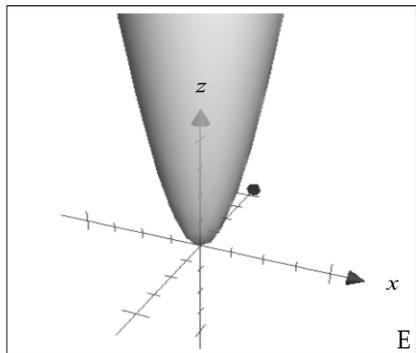
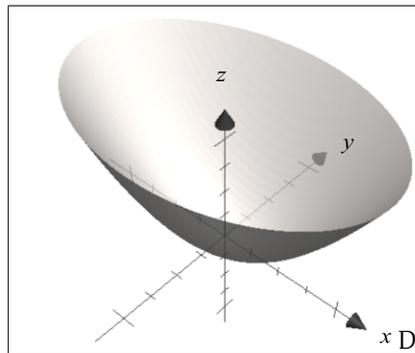
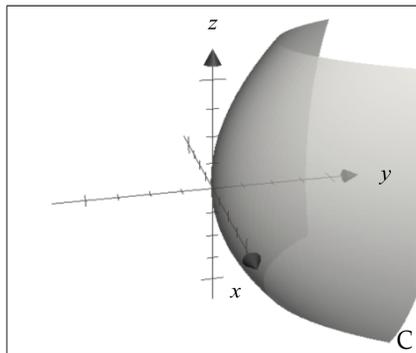
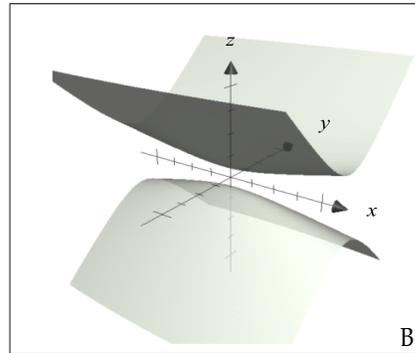
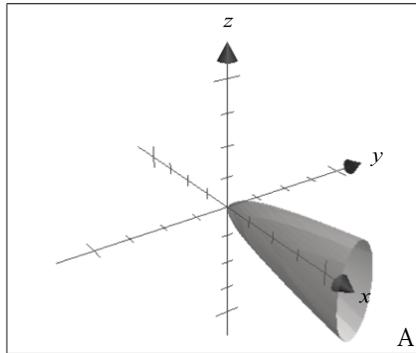
$$z^2 = x - 9y^2.$$

(a) Sketch the traces of S in the $x = 4$ and $z = 0$ planes, labeling your axes carefully.

(3 + 3 = 6 points)

(b) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?

(4 points)



5. (Extra credit) Find a suitable function $f(x)$ and interval $[a, b]$ so that

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{3}{n} \cos\left(\frac{k}{n}\right).$$

(10 points)

Formula Sheet

You may refer to these formulae if necessary.

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}.$$