The University of British Columbia

Midterm 1 - February 3, 2012

Mathematics 105, 2011W T2

Section 203

Closed book examination | Time: 50 minutes

Last Name ____________________ **First** _____________ **SID** __________________

Instructor name: Keqin Liu

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in"calculator-ready" form. Simplification of the final answer is worth at most one point.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Q	Points	Max
1		50
2		20
3		20
4		10
5 (extra credit)		10
Total		100

1. (a) You are given a function $f(x, y)$ with the following properties:

$$f(1,2) = 0, \quad f_y(1,2) = -10, \quad f(1.2, 1.9) = -1.$$

Use this information to estimate $f_x(1, 2)$.

(8 points)

(b) Let $\mathbf{v} = \langle 1, 2, -3 \rangle$ and $\mathbf{w} = \langle 4, 0, 1 \rangle$. Are the two vectors $\mathbf{v}$ and $\mathbf{w}$ parallel, perpendicular or neither? Justify your answer.

(8 points)

(c) Find a unit vector parallel to $\langle 3, -2, \sqrt{3} \rangle$.

(8 points)

(d) A function $f(x)$ is defined on the interval $[-2, 4]$ as follows:

$$f(-2) = f(2) = f(4) = 0, \quad f(0) = 4, \quad f(3) = -1,$$

and the graph of f consists of straight line segments joining these points. Compute the value of the integral

$$\int_{-2}^{4} f(x)\,dx.$$

(8 points)

(e) You are given a function f which has continuous partial derivatives of all orders and for which $f_{xy}(x, y) = x^2y + xy^2$. Use this information to find f_{yxy}. State clearly any result that you use.

(8 points)

(f) Let R be the semicircular region $\{x^2 + y^2 \leq 9,\ y \geq 0\}$. Find the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 - 4y$$

on the *boundary of the region* R.

(10 points)

2. Find *all* critical points of the function

$$f(x, y) = 3x^2 - 6xy + y^3 - 9y.$$

Classify each point as a local maximum, local minimum, or saddle point.

(10 + 10 = 20 points)

3. A company wishes to build a new warehouse. It should be situated on the northeast quarter of the Oval, an expressway whose shape is given by the equation

$$\frac{x^2}{9} + \frac{y^2}{16} = 1.$$

Here x and y are measured in kilometers. From the company's point of view, the desirability of a location on the Oval is measured by the sum of its horizontal and vertical distances from the origin. The larger the sum is, the more desirable the location is. Find using the method of Lagrange multipliers the location on the Oval that is most desirable to the company.

Clearly state the objective function and the constraint. There is no need to justify that the solution you obtained is the absolute max or min. **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**

(20 points)

4. Consider the surface S given by

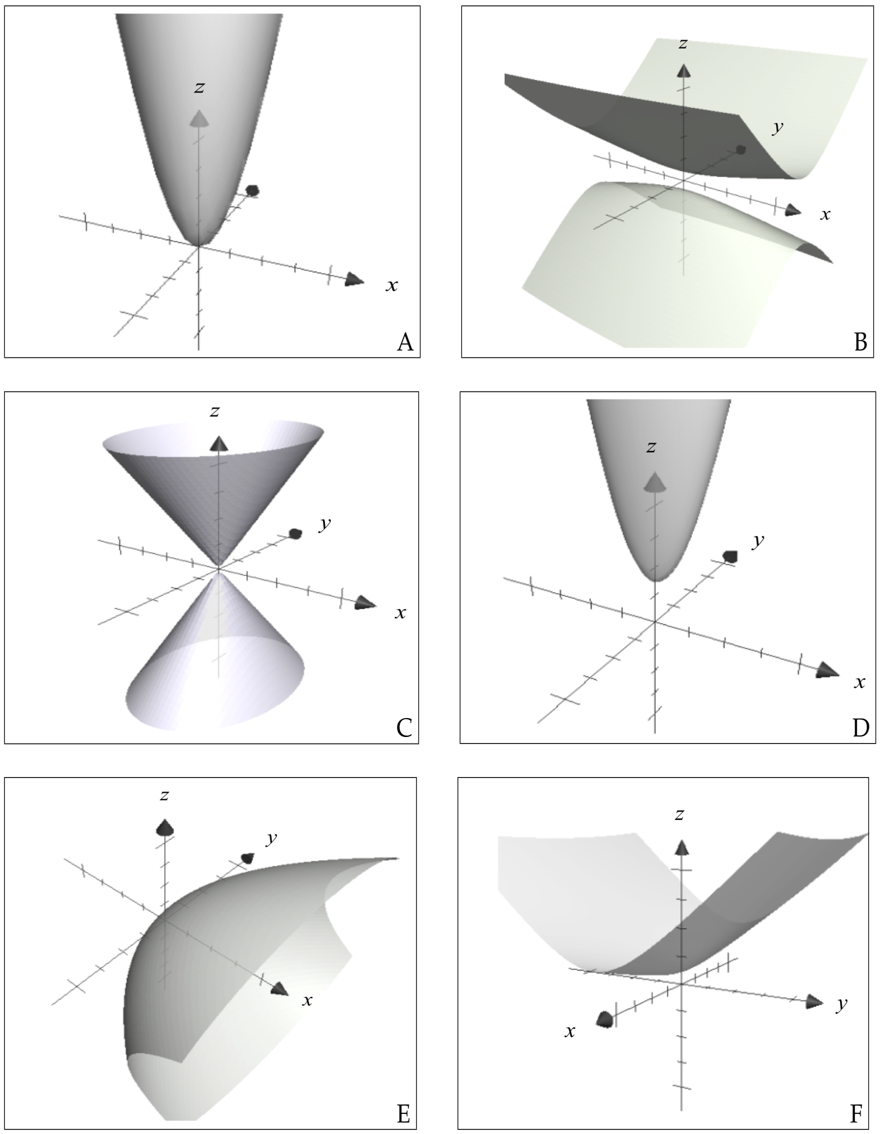
$$x - \frac{y^2}{16} = \frac{z^2}{9}.$$

(a) Sketch the traces of S in the $z = 0$ and $x = 1$ planes.

(3 + 3 = 6 points)

(b) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?

(4 points)



5. (Extra credit) Transform the limit of the following Riemann sum to a definite integral:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\frac{1}{1+\left(\frac{k}{n}\right)^2}.$$

(10 points)

Formula Sheet

You may refer to these formulae if necessary.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$