MATH 105 Practice Problem Set 2 Questions

1. Parts (a)–(d) are `TRUE` or `FALSE`, **plus explanation**. Give a full-word answer `TRUE` or `FALSE`. If the statement is true, explain why, using concepts and results from class to justify your answer. If the statement is false, give a counterexample.

   (a) [5 marks] The level curves of the plane $ax + by + cz = d$, where $a, b, c, d \neq 0$, are parallel lines in the xy-plane.

   (b) [5 marks] The domain of the function $g(x, y) = \ln\left((x+1)^2 + (y-2)^2 - 1\right)$ consists of all points (x, y) lying strictly in the interior of a circle centered at $(-1, 2)$ of radius 1.

   (c) [5 marks] There exists a function $f(x, y)$ defined on $\mathbb{R}^2$ such that $f_x(x, y) = \cos(3y)$ and $f_y(x, y) = x^4$.

   (d) [5 marks] If $f(x, y)$ is a function such that $f_x(x, y) = 2x + y$ and $f_y(x, y) = x + 1$, then f is differentiable at every point in $\mathbb{R}^2$.

2. Consider the surface $zx^2 = z^2 - y^2$.

   (a) [10 marks] Find the equations and sketch the level curves for $z = -1, 0, 1$ on the same set of axes.

   (b) [5 marks] Find all values of z which correspond to a level curve containing the point $(x, y) = (2, 0)$.

3. 10 marks Show that $u(x,t) = t^{-\frac{1}{2}} e^{-\frac{x^2}{4t}}$ is a solution to the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

4. Consider the Body Mass Index function being calculated by $b(w,h) = \frac{w}{h^2}$, where w is the weight in kilograms and h is the height in meters.

   (a) [5 marks] Compute b_w and b_h.

   (b) [5 marks] For a fixed weight, as the height increases, how does the Body Mass Index change? Explain using the answers in part a.

5. Let $f(x, y) = y^3 \sin(4x)$.

   (a) $\boxed{\text{5 marks}}$ Explain in your own words what it means for the function $f(x, y)$ to be differentiable at a point (a, b).

   (b) $\boxed{\text{5 marks}}$ Show that f is differentiable at every point in $\mathbb{R}^2$.

6. 15 marks Find the maximum and minimum values of the function $f(x, y) = ye^x - e^y$ in the area bounded by the triangle whose vertices are $(4, 1)$, $(1, 1)$ and $(4, 4)$.

7. 10 marks Find the maximum and minimum of $f(x, y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$.

8. Let $f(x) = 2x + 1$.

   (a) $\boxed{\text{5 marks}}$ Write down the left Riemann sum for $\int_1^5 f(x)\,dx$.

   (b) $\boxed{\text{10 marks}}$ Compute the limit as $n \to \infty$ of the Riemann-sum expression found in part (a) and thus evaluate $\int_1^5 f(x)\,dx$.