The University of British Columbia

Midterm 2 - March 16, 2012

Mathematics 105, 2011W T2

Section 207

Closed book examination Time: 50 minutes

Last Name ________ **First** ________ **SID** ________

Instructor name: Michael Lindstrom

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form. Simplification of the final answer is worth at most one point.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Q	Points	Max
1		60
2		20
3		20
4 (extra credit)		5
Total		100

1. (a) Find the derivative of the function

$$f(x) = \int_1^{\ln x} \sqrt{4 + 2z + \sin z}\, dz$$

at the point $x = 1$.

(10 points)

(b) Use Simpson's rule to approximate

$$\int_0^{\pi} \sin x \, dx$$

with $n = 4$ subintervals. Find a bound on the error. **No need to simplify your answers!**

(5 + 5 = 10 points)

(c) Find the definite integral

$$\int_{-2}^{1} \frac{5}{x^3}\, dx.$$

(10 points)

(d) A fair coin is tossed three times. Find the probability density function for the number of heads obtained in this experiment.

(10 points)

(e) Find the indefinite integral

$$\int \cos^5 \theta \, d\theta.$$

(10 points)

(f) Solve the initial value problem

$$y' = 2te^y + e^y, \qquad y(0) = 1.$$

(10 points)

2. Evaluate the definite integral:

$$\int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx.$$

(20 points)

3. At a certain supermarket the amount of time one must wait at the express lane is a random variable with density function

$$f(x) = \frac{k}{(x+1)^2}, \qquad 0 \leq x \leq 10.$$

(10+10 = 20 points)

(a) Find the value of k.

(b) What is the expected waiting time at the express checkout at the supermarket?

4. (Extra credit) The population size in a city evolves as follows. The birth rate (i.e., number of births/year) is 3.5% of the existing population size, while the death rate is 2.5% of the same. In addition, 1000 people move into the city every year, and 3000 migrate out every year. Assume that all events are spread evenly throughout the year. The city's current population is 100,000. Write down an initial value problem that models the evolution of the population size as a function of time. **Do not solve this problem!**

(5 points)

Formula Sheet

You may refer to these formulae if necessary.

Trigonometric formulae:

$$\cos^2 x = \frac{1+\cos(2x)}{2}.$$
$$\sin^2 x = \frac{1-\cos(2x)}{2}.$$

Simpson's rule:

$$S_n = \frac{\Delta x}{3}\Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 4f(x_{n-1}) + f(x_n)\Big).$$
$$E_s = \frac{K(b-a)(\Delta x)^4}{180}, \quad |f^{(4)}(x)| < K \text{ on } [a,b].$$

Indefinite Integrals:

$$\int \sec x \, dx = \ln\big|\sec x + \tan x\big| + C.$$

Probability:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\, dx.$$
$$\mathrm{Var}[X] = \int_{-\infty}^{\infty} (x - \mathbb{E}[X])^2 f(x)\, dx.$$