Math 121 Assignment 6

Due Friday March 4

1. If 100 N.cm of work must be done to compress an elastic spring to 3 cm shorter than its normal length, how much work must be done to compress it 1 cm further? Recall that by Hooke's law, the force required to compress an elastic spring to x units shorter than its natural length is proportional to x.

2. A bucket is raised vertically from ground level at a constant speed of 2m/min by a winch. If the bucket weighs 1 kg and contains 15 kg of water when it starts up but loses water by leakage at the rate of 1 kg/min thereafter, how much work must be done by the winch to raise the bucket to a height of 10m?

3. For each of the following equations, find a function $y(x)$ that obeys it.
   (a) $y(x) = 3 + \int_0^x e^{-y(t)}\,dt$.
   (b) $x^2y' + y = x^2e^{1/x}$, $y(1) = 3e$.

4. Find the equation of a curve that passes through the point $(2,4)$ and has slope $3y/(x-1)$ at any point on it.

5. The initial balance in the account was \$1000. Interest is paid continuously into the account at a rate of 10% per annum, compounded continuously. The account is also being continuously depleted by taxes at the rate of $y^2 10^{-6}$ dollars per year, where $y = y(t)$ is the balance in the account after t years. How large can the account grow? How long will it take the account to grow to half its balance?

6. Identify the parametric curves
$$\begin{cases} x = \cosh t \\ y = \sinh^2 t \end{cases} \quad \text{and} \quad \begin{cases} x = \cos t + \sin t, \\ y = \cos t - \sin t. \end{cases}$$

7. For the following two examples, determine the points where the given parametric curves have horizontal and vertical tangents.
$$\begin{cases} x &= \frac{4}{1+t^2} \\ y &= t^3 - 3t \end{cases} \quad \text{and} \quad \begin{cases} x &= t^3 - 3t, \\ y &= t^3 - 12t. \end{cases}$$

8. Find the length of the curve $x = e^t - t$, $y = 4e^{t/2}$ from $t = 0$ to $t = 2$.

9. Sketch the polar graph of the equation $r = 1 + 2\cos 2\theta$ and find the area of one of the two smaller loops.

10. Find the area of the region inside the cardioid $r = 1 + \cos\theta$ and to the left of the line $x = 1/4$.

11. Show that a plane that is not parallel to the axis of a circular cylinder intersects the cylinder in an ellipse.

12. At what points do the curves $r^2 = 2\sin 2\theta$ and $r = 2\cos\theta$ intersect? At what angle do the curves intersect at each of these points?

13. A tractrix is a curve in the first quadrant of the (x, y) plane, starting from the point $(L, 0)$, and having the property that if the tangent line to the curve at P meets the y-axis at Q, then the length of PQ is the constant L. (For example, think of a trailer of length L attached to a tractor which is sitting at the origin. The rear end P of the trailer was originally lying at $(L, 0)$. As the tractor moves away along the y-axis, the path traced out by P is a tractrix.) Find the equation of the tractrix.