Introduction to Harmonic Analysis - Math 541

Spring 2016

- ***Instructor: Malabika Pramanik***
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- ***Office hours: To be announced.***

• **Web page:** The course website is

http://www.math.ubc.ca/~malabika/teaching/ubc/spring16/math541/index.html

Homework assignments and all relevant course information (such as changes to office hours if any, or solutions to homework problems if needed) will be posted here.

• **Text:** There are no required textbooks. The following textbooks are recommended.

– *Lectures on Harmonic Analysis (2003), by T.H. Wolff*, AMS, ISBN: 978-0-8218-3449-7.

– *An Introduction to Harmonic Analysis (3rd edition), by Y. Katznelson*, Cambridge, ISBN: 978-0-521-54359-2.

– *Singular Integrals and Differentiability Properties of Functions (1970), by E. Stein*, Princeton University Press, ISBN: 0-691-08079-8.

– *Introduction to Fourier Analysis on Euclidean Spaces (1971), by E. Stein, and G. Weiss*, Princeton University Press, ISBN: 0-691-08078-X.

– *Harmonic Analysis: Real-variable Methods, Orthogonality and Oscillatory Integrals (1993), by E. Stein*, Princeton University Press, ISBN: 0-691-03216-5.

– *Classical and Modern Fourier Analysis, by L. Grafakos.*

• **Course outline :** The core topics of the course are the following:

1. *Basic material concerning Fourier series, Fourier transform and Fourier inversion*
   - Fourier basis for $L^2(\mathbb{T})$
   - Convolution
   - Approximate identities
   - Temperate distributions
   - Some applications
2. *Convergence of Fourier series*
   - Decay of Fourier coefficients
   - Uniform convergence of Fourier series

- Pointwise convergence and almost everywhere divergence
- Norm convergence

3. *Interpolation of operators*
   - Complex methods (Riesz-Thörin theorem, analytic interpolation)
   - Real methods (Marcinkiewicz interpolation theorem)
   - Applications (Hausdorff-Young inequality, Young's convolution inequality, fractional integration, Hardy-Littlewood maximal theorem).
4. *Singular integral operators*
   - Calderón-Zygmund kernels
   - Some multiplier operators
   - The Calderón-Zygmund decomposition
   - L^p boundedness of Calderón-Zygmund singular integral operators
   - Homogeneous distributions, Hilbert transform, Riesz transform.
5. *Littlewood-Paley theory*
   - Almost orthogonality in Hilbert spaces, Cotlar-Knapp-Stein lemma
   - A square function that characterizes L^p
   - Variations and applications

Time permitting, we will also consider other special topics.

• **Lectures :** Monday, Wednesday, Friday 11 am - 12 noon in Mathematics Annex 1118.

• **Grading Policy :** Homework problems will be posted regularly on the course website. In addition, you will be required to give a presentation in class on a topic relevant to the course material and agreed upon by yourself and the instructor. Your total score will be a weighted average of your homework and in-class presentation, with the breakdown as follows.

Homework	75%
Presentation	25%