Inverting the Laplace transform

The inverse Laplace transform of

$$\frac{1-2s}{s^2+4s+5}$$

is

A. $-2e^{-2t}\cos t - 5e^{-2t}\sin t$

B. $-2e^{t}\cos 2t + 5e^{t}\sin 2t$

C. $-2e^{2t}\cos t + 5e^{2t}\sin t$

D. $-2e^{-2t}\cos t + 5e^{-2t}\sin t$

E. $2e^{-2t}\cos t - 5e^{-2t}\sin t$

Solving an IVP using Laplace transforms

Use the Laplace transform to solve the initial value problem

$$y'' + \omega^2 y = \cos(2t), \qquad \omega^2 \neq 4, \quad y(0) = 1, \quad y'(0) = 0.$$

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Answer: $(\omega^2 - 4)^{-1}\left[(\omega^2 - 5)\cos(\omega t) + \cos(2t)\right]$.

Translations and Scalings

$$\mathcal{L}^{-1}[F(\alpha s + \beta)] =$$

A. $\alpha^{-1} e^{-\frac{\beta}{t\alpha}} f\left(\frac{t}{\alpha}\right)$

B. $\alpha^{-1} e^{-\frac{\beta t}{\alpha}} f\left(\frac{t}{\alpha}\right)$

C. $\alpha e^{-\frac{\beta t}{\alpha}} f\left(t\alpha\right)$

D. $\alpha e^{-\alpha\beta t} f\left(t\alpha\right)$

The Heaviside function

Express

$$f(t) = \begin{cases} t & \text{if } 0 \le t < 1 \\ t-1 & \text{if } 1 \le t < 2 \\ t-2 & \text{if } 2 \le t < 3 \\ 0 & \text{if } t \ge 3. \end{cases}$$

in terms of the Heaviside function.

A. $f(t) = -u_1(t) - u_2(t) - u_3(t)(t-2)$.

B. $f(t) = t - u_1(t) - u_2(t) - u_3(t)$.

C. $f(t) = t - u_1(t) - u_2(t) - u_3(t)(t-2)$.

D. $f(t) = t - u_1(t) + u_2(t) - u_3(t)(t-2)$.

E. $f(t) = t + u_1(t) - u_2(t) - u_3(t)(t-2)$.

Impulse Functions

Find the solution of the initial value problem

$$y'' + 2y' + 2y = \delta(t - \pi), \qquad y(0) = 1, \quad y'(0) = 0.$$

Answer: $y = e^{-t}\cos t + e^{-t}\sin t + u_\pi(t)e^{-(t-\pi)}\sin(t - \pi).$