Method of undetermined coefficients

Consider the system

$$\mathbf{x}' = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix} \mathbf{x} + e^t \mathbf{a}$$

where $\mathbf{a}$ is an arbitrary constant vector.

Which of the following possibilities would you choose as the most economic $\mathbf{x}_p$ using the method of undetermined coefficients?

A. $\mathbf{x}_p = \mathbf{u}e^t$

B. $\mathbf{x}_p = \mathbf{u}te^t + \mathbf{v}e^t$

C. $\mathbf{x}_p = \mathbf{u}(te^t + e^t)$

D. $\mathbf{x}_p = \mathbf{u}te^t$

E. $\mathbf{x}_p = \mathbf{u}t^2e^t + \mathbf{v}te^t + \mathbf{w}e^t$

Undetermined coefficients (ctd)

Consider the system in the previous example.

- Suppose that $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Show that $\mathbf{x}_p = \mathbf{a}te^t$.
- Suppose that $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Show that $\mathbf{x}_p$ cannot be of the form $\mathbf{u}te^t$.
- Now show that for $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\mathbf{x}_p = \begin{pmatrix} 4 \\ -2 \end{pmatrix} te^t + \begin{pmatrix} -3 \\ 0 \end{pmatrix} e^t.$$

What is the source of the disparity in the forms for $\mathbf{x}_p$ in the two examples above?

Critical points, linearizations and stability

How many critical points does the following system have?

$$x' = (2+x)(y-x), \qquad y' = (4-x)(y+x).$$

A. 0

B. 1

C. 2

D. 3

E. 4

Find the linearizations of each of the critical points, and use it to identify the nature of the critical points.