A non-constant coefficient example

Find the general solution of the equation $x^2y'' - 3xy' + 4y = x^2 \ln x$

Answer: $C_1x^2 + C_2x^2 \ln x + \frac{1}{6}x^2(\ln x)^3$.

Method of undetermined coefficients

A particular solution of the equation

$$y'' - 4y' + 4y = 2t^2 + 4te^{2t} + t\sin 2t$$

has to be of the form

Method of undetermined coefficients

A particular solution of the equation

$$y'' - 4y' + 4y = 2t^2 + 4te^{2t} + t\sin 2t$$

has to be of the form

A. $A_2t^2 + (B_1t + B_0)e^{2t} + (C_1t + C_0)\cos 2t + (D_1t + D_0)\sin 2t.$

B. $(A_2t^2 + A_1t + A_0) + (B_1t + B_0)e^{2t} + (D_1t + D_0)\sin 2t.$

C. $(A_2t^2 + A_1t + A_0) + (B_3t^3 + B_2t^2)e^{2t} + (C_1t + C_0)\cos 2t + (D_1t + D_0)\sin 2t.$

D. $(A_2t^2 + A_1t + A_0) + (B_1t + B_0)e^{2t} + C_2t^2\sin 2t + D_2t^2\cos 2t.$

E. $(A_2t^2 + A_1t + A_0)e^{2t} + C_2t^2\sin 2t + D_2t^2\cos 2t.$

Method of undetermined coefficients

A particular solution of the equation

$$y'' - 4y' + 4y = 2t^2 + 4te^{2t} + t\sin 2t$$

has to be of the form

A. $A_2t^2 + (B_1t + B_0)e^{2t} + (C_1t + C_0)\cos 2t + (D_1t + D_0)\sin 2t.$

B. $(A_2t^2 + A_1t + A_0) + (B_1t + B_0)e^{2t} + (D_1t + D_0)\sin 2t.$

C. $(A_2t^2+A_1t+A_0)+(B_3t^3+B_2t^2)e^{2t}+(C_1t+C_0)\cos 2t+(D_1t+D_0)\sin 2t.$

D. $(A_2t^2 + A_1t + A_0) + (B_1t + B_0)e^{2t} + C_2t^2\sin 2t + D_2t^2\cos 2t.$

E. $(A_2t^2 + A_1t + A_0)e^{2t} + C_2t^2\sin 2t + D_2t^2\cos 2t.$

Find the general solution of the given equation.

Decoupling nonhomogeneities and initial conditions

Show that the solution to the initial value problem

$$L[y] = y" + p(t)y' + q(t)y = g(t), \qquad y(t_0) = a_0,\ y'(t_0) = a_1$$

can be written as the sum of two functions u and v, where

- u solves the homogeneous equation $L[u] = 0$ with $u(t_0) = a_0$ and $u'(t_0) = a_1$
- v solves the inhomogeneous equation $L[v] = g(t)$, but with initial conditions $v(t_0) = 0,\ v'(t_0) = 0$.

Forced vibration

A mass that weighs 8 lbs stretches a spring 6 in. The system is acted on by an external force of $8 \sin 8t$ lb. If the mass is pulled down 3 in and then released, determine the position of the mass at any time.

Forced vibration

A mass that weighs 8 lbs stretches a spring 6 in. The system is acted on by an external force of $8 \sin 8t$ lb. If the mass is pulled down 3 in and then released, determine the position of the mass at any time.

Answer: $\frac{1}{4} \cos 8t + \frac{1}{4} \sin 8t - 2t \cos 8t$

New solutions from old

Let L be a second-order linear ODE with continuous coefficients, whose coefficient functions are unknown. A minimum of how many solutions to the inhomogeneous equation $L[y] = g(t)$ (with possibly varying initial data based at t_0) do you need to know in order to specify the general solution of $L[y] = 0$?

New solutions from old

Let L be a second-order linear ODE with continuous coefficients, whose coefficient functions are unknown. A minimum of how many solutions to the inhomogeneous equation $L[y] = g(t)$ (with possibly varying initial data based at t_0) do you need to know in order to specify the general solution of $L[y] = 0$?

New solutions from old

Let L be a second-order linear ODE with continuous coefficients, whose coefficient functions are unknown. A minimum of how many solutions to the inhomogeneous equation $L[y] = g(t)$ (with possibly varying initial data based at t_0) do you need to know in order to specify the general solution of $L[y] = 0$?

A. cannot be determined.

B. 1

C. 2

D. 3

E. 4

New solutions from old

Let L be a second-order linear ODE with continuous coefficients, whose coefficient functions are unknown. A minimum of how many solutions to the inhomogeneous equation $L[y] = g(t)$ (with possibly varying initial data based at t_0) do you need to know in order to specify the general solution of $L[y] = 0$?

A. cannot be determined.

B. 1

C. 2

D. 3

E. 4

Write down a set of initial value problems for the inhomogeneous equation that would lead to the general solution of the homogeneous equation.

New solutions from old (ctd)

What would your answer be if L is a *known* differential operator? In other words,

$$L[y] = y" + p(t)y' + q(t)y,$$

where p, q and $g \not\equiv 0$ are known continuous functions on an interval I containing t_0.

New solutions from old (ctd)

What would your answer be if L is a *known* differential operator? In other words,

$$L[y] = y" + p(t)y' + q(t)y,$$

where p, q and $g \not\equiv 0$ are known continuous functions on an interval I containing t_0.

A. cannot be determined.

B. 1

C. 2

D. 3

E. 4