Errata for Math Club Math 100/180 Package

December 13, 2010

Dec. 2003, 1(b): Can be solved by factoring the denominator; l'Hôpital's rule is not in the syllabus.

Dec. 2003, 1(c): Solution without l'Hôpital's rule: We have $\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{2\sin(\theta)\cos(\theta)}{\cos(2\theta)}$ so $\frac{\sin(\theta)}{\tan(2\theta)} = \frac{\cos(2\theta)}{2\cos(\theta)}$. Now both the numerator and denominator are continuous function and the denominator is non-zero at $\theta = 0$, so the limit is $\frac{\cos(0)}{2\cos(0)} = \frac{1}{2}$.

Dec. 2003, 5: "Maxima" is properly only the plural of "maximum". What the solution is trying to say is that $f(x)$ achieves its maximum when $x = \frac{1}{k}$.

Dec. 2005, 1(k): Can be solved exactly so no need to make linear approximation: $v(t) = \frac{ds}{dt}$ so $s(t) = \frac{2}{3}t^{3/2} + C$. At $t = 9$ we have $20 = \frac{2}{3}9^{3/2} + C$ so $C = 20 - \frac{2}{3}27 = 2$ and $s(10) = \frac{2}{3}10^{3/2} + 2$.

Dec. 2005, 2: Solve using related rates; height of the passenger relative to the center of the wheel is $y = 10\sin\theta$, so rate of change of the height is $\frac{dy}{dt} = 10\cos\theta \cdot \frac{d\theta}{dt} = 100\cos\theta\frac{\text{rad}}{\text{min}}$. At the given moment we have $\sin\theta = \frac{6}{10}$ so $\cos\theta = \pm\sqrt{1 - \frac{36}{100}} = \pm\sqrt{\frac{64}{100}} = \pm 0.8$. It follows that at the given moment we have

$$\frac{dy}{dt} = 100 \cdot 0.8\frac{\text{rad}}{\text{min}} = 80\frac{\text{rad}}{\text{min}}$$

where we took the positive root since we are given that the passenger is rising.

Dec. 2007, 1(f): "Noting that" has a spurious exponent.

Dec. 2008, 1(l): The derivative of $\arcsin u$ is $\frac{1}{\sqrt{1-u^2}}$ and not as written.

Dec. 2008, 7: We were actually told that $f(x) = 2$ at $x = 0$.

Dec. 2009, 1(c): l'Hôpital's rule is not in the syllabus. Two alternative solutions:

$$\lim_{x\to 0}\frac{\sin(4x)}{x} = \lim_{x\to 0}\frac{\sin(4x) - \sin(0)}{x - 0} \stackrel{\text{def}}{=} \left[\frac{d}{dx}(\sin(4x))\right]_{x=0} = [4\cos(4x)]_{x=0} = 4$$

and

$$\lim_{x\to 0}\frac{\sin(4x)}{x} = 4\lim_{x\to 0}\frac{\sin(4x)}{4x} = 4\lim_{u\to 0}\frac{\sin u}{u} = 4 \cdot 1 = 4\,.$$

(The last limit was postulated in class in section 105).