Name (print):

Student number:

Date: *February 11, 2016*

Time: *6:00 p.m. to 7:30 p.m.*

Number of pages: *10 (including cover page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

For examiners' use only		
Question	Mark	Possible marks
1		20
2		5
3		7
4		7
5		5
6		1
Total		44

In your answers, you must simplify standard trigonometric values (*e.g.* $\cos\left(\frac{\pi}{3}\right)$) and exponential values (*e.g.* $\log\left(e^3\right)$). Otherwise, it is sufficient for your answers to be in "calculator-ready" form.

In general, you may use any result proven in class or on assignments. You may find the following identities helpful:

$$\begin{aligned}
\sin^2(\theta) + \cos^2(\theta) &= 1 \\
\tan^2(\theta) + 1 &= \sec^2(\theta).
\end{aligned}$$

This page may be used for rough work. It will not be marked.

1. *You must receive a passing grade on this question in order to pass the course. However, if you do not receive a passing grade on this question, you will be given more chances at future dates to rewrite it, even though your original grade will remain.*

   (a) [**3 marks**] Evaluate $\displaystyle\int_1^e \frac{\log(t)}{t}\,dt.$

   (b) [**3 marks**] Evaluate $\displaystyle\int_0^1 \frac{t^3}{t^2+1}\,dt.$

(c) [**4 marks**] Evaluate $\displaystyle\int_2^\infty \frac{t-1}{t^{5/2}}\,dt.$

(d) [**3 marks**] Evaluate $\displaystyle\int_0^{-\pi/3} \sin^4(t)\cos^3(t)\,dt.$

(e) **[3 marks]** Evaluate $\displaystyle\int_{-1}^{1} \frac{5t}{t^2 - t - 6}\, dt$.

(f) **[4 marks]** Evaluate $\displaystyle\int_{1}^{\sqrt{3}} \frac{1}{t^2\sqrt{t^2 + 1}}\, dt$.

2. (a) [**1 mark**] Define, using Riemann sums, what it means for a function $f(t)$ to be integrable on the interval $[0, 1]$.

(b) [**4 marks**] Suppose $f(t)$ is integrable and negative on the interval $[0, 1]$. Let

$$g(t) = \begin{cases} f(t) & \text{if } 0 \leq t < 1 \\ 2 & \text{if } t = 1 \end{cases}.$$

Prove that $g(t)$ is integrable on $[0, 1]$.

3. Let $f(t)$ be the solution to the integral equation $f(t) = 2 - \int_0^{\sqrt{t}} 4sf\left(s^2\right)\, ds$.

(a) [**3 marks**] Find $f(t)$.

(b) [**4 marks**] Evaluate $\int_0^\infty 2tf\left(t^2\right)\, dt$.

4. Let $f(t)$ be a function such that $f''(t)$ is continuous and positive for all t.

(a) [**2 marks**] Prove that $\displaystyle\int_0^{2\pi} f''(t) \cos(t)\, dt \le f'(2\pi) - f'(0)$.
(Hint: use the fact that $|\cos(t)| \le 1$.)

(b) [**4 marks**] Prove that $\displaystyle\int_0^{2\pi} f(t) \cos(t)\, dt = f'(2\pi) - f'(0) - \int_0^{2\pi} f''(t) \cos(t)\, dt$.

(c) [**1 mark**] Explain why it follows that $\displaystyle\int_0^{2\pi} f(t) \cos(t)\, dt \ge 0$. You may use the results of part (a) and part (b) without proof.

5. Let S be the solid obtained by rotating the shaded area below about the y-axis. Find the volume of S.
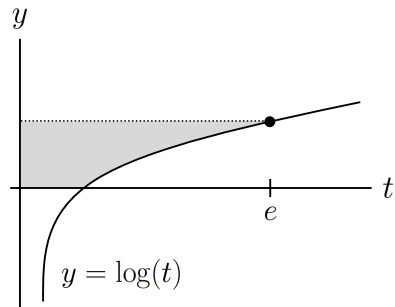
(a) [**2 marks**] Write down an integral describing the volume of S.

(b) [**3 marks**] Evaluate the integral in part (a).

6. [**1 bonus mark**] Write a short poem about your love of mathematics.