

Name (print):

Student number:



University of British Columbia
MATH 101 (Vantage): Midterm test

Date: *February 11, 2016*

Time: *6:00 p.m. to 7:30 p.m.*

Number of pages: *10 (including cover page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For examiners' use only		
Question	Mark	Possible marks
1		20
2		5
3		7
4		7
5		5
6		1
Total		44

In your answers, you must simplify standard trigonometric values (*e.g.* $\cos\left(\frac{\pi}{3}\right)$) and exponential values (*e.g.* $\log(e^3)$). Otherwise, it is sufficient for your answers to be in “calculator-ready” form.

In general, you may use any result proven in class or on assignments. You may find the following identities helpful:

$$\begin{aligned}\sin^2(\theta) + \cos^2(\theta) &= 1 \\ \tan^2(\theta) + 1 &= \sec^2(\theta).\end{aligned}$$

This page may be used for rough work. It will not be marked.

1. *You must receive a passing grade on this question in order to pass the course. However, if you do not receive a passing grade on this question, you will be given more chances at future dates to rewrite it, even though your original grade will remain.*

(a) **[3 marks]** Evaluate $\int_1^e \frac{\log(t)}{t} dt$.

(b) **[3 marks]** Evaluate $\int_0^1 \frac{t^3}{t^2 + 1} dt$.

(c) [4 marks] Evaluate $\int_2^{\infty} \frac{t-1}{t^{5/2}} dt$.

(d) [3 marks] Evaluate $\int_0^{-\pi/3} \sin^4(t) \cos^3(t) dt$.

(e) [3 marks] Evaluate $\int_{-1}^1 \frac{5t}{t^2 - t - 6} dt$.

(f) [4 marks] Evaluate $\int_1^{\sqrt{3}} \frac{1}{t^2 \sqrt{t^2 + 1}} dt$.

2. (a) [**1 mark**] Define, using Riemann sums, what it means for a function $f(t)$ to be integrable on the interval $[0, 1]$.

(b) [**4 marks**] Suppose $f(t)$ is integrable and negative on the interval $[0, 1]$. Let

$$g(t) = \begin{cases} f(t) & \text{if } 0 \leq t < 1 \\ 2 & \text{if } t = 1 \end{cases} .$$

Prove that $g(t)$ is integrable on $[0, 1]$.

3. Let $f(t)$ be the solution to the integral equation $f(t) = 2 - \int_0^{\sqrt{t}} 4sf(s^2) ds$.

(a) **[3 marks]** Find $f(t)$.

(b) **[4 marks]** Evaluate $\int_0^{\infty} 2tf(t^2) dt$.

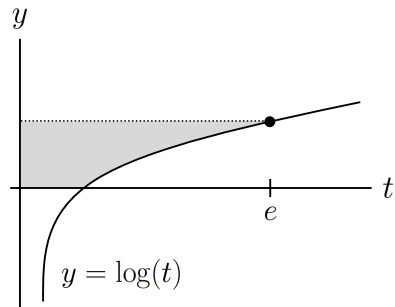
4. Let $f(t)$ be a function such that $f''(t)$ is continuous and positive for all t .

(a) [**2 marks**] Prove that $\int_0^{2\pi} f''(t) \cos(t) dt \leq f'(2\pi) - f'(0)$.
(Hint: use the fact that $|\cos(t)| \leq 1$.)

(b) [**4 marks**] Prove that $\int_0^{2\pi} f(t) \cos(t) dt = f'(2\pi) - f'(0) - \int_0^{2\pi} f''(t) \cos(t) dt$.

(c) [**1 mark**] Explain why it follows that $\int_0^{2\pi} f(t) \cos(t) dt \geq 0$. You may use the results of part (a) and part (b) without proof.

5. Let S be the solid obtained by rotating the shaded area below about the y -axis. Find the volume of S .



- (a) [**2 marks**] Write down an integral describing the volume of S .

- (b) [**3 marks**] Evaluate the integral in part (a).

6. [1 **bonus mark**] Write a short poem about your love of mathematics.