Name (underline your surname):

Student number:

University of British Columbia

MATH 101 (Vantage): Midterm test

Date: *February 16, 2017*

Time: *6:00 p.m. to 7:30 p.m.*

Number of pages: *11 (including cover page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For examiners' use only		
Question	Mark	Possible marks
1		15
2		5
3		6
4		5
5		4
6		5
7		1 (bonus)
Total		40

Note that your answers must be in “calculator-ready” form, but they do not have to be simplified.

In general, you may use any result proven in class or on assignments. You have to use the Riemann sum definition of integral only in question 2. You may use without proof the formulas

$$\begin{aligned}\sin^2(\theta) &= \tfrac{1}{2}\left(1-\cos(2\theta)\right),\\ \cos^2(\theta) &= \tfrac{1}{2}\left(1+\cos(2\theta)\right).\end{aligned}$$

This page may be used for rough work. It will not be marked.

1. (a) **[3 marks]** Evaluate $\displaystyle\int_1^2 (t+5)(t-1)^{10}\,dt$.

(b) **[3 marks]** Evaluate $\displaystyle\int_1^e t\log(4t)\,dt$.

(c) **[3 marks]** Evaluate $\displaystyle\int_0^\infty te^{-t^2/5}\,dt$.

(d) **[3 marks]** Evaluate $\displaystyle\int_3^4 \frac{t^2+4t+12}{(t-2)\,(t^2+4)}\,dt$.

(e) **[3 marks]** Evaluate $\displaystyle\int_0^{\pi/4} \frac{\sin^3(\theta)}{\cos^2(\theta)}\, d\theta$.

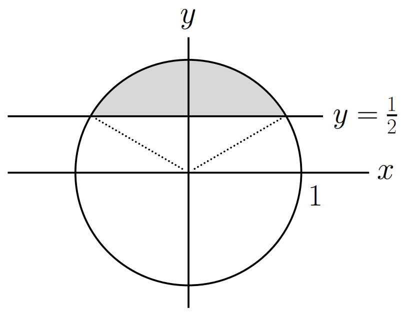
2. (a) **[1 mark]** Define, using Riemann sums, what it means for a function $f(t)$ to be integrable on an interval $[l, r]$.

(b) **[4 marks]** Let

$$f(t) = \begin{cases} A & \text{if } t = k \\ B & \text{otherwise.} \end{cases},$$

where A, B and k are constants with $A < B$. Prove that $f(t)$ is integrable on any finite interval $[l, r]$, where $l < k < r$.

3.



(a) **[4 marks]** Use techniques of integration to calculate the area of the unit circle lying above the line $y = \frac{1}{2}$. (This area is shaded above.)

(b) **[2 marks]** Verify your solution in part (a) by calculating the area without using calculus. (Hint: what is the area of the triangle below the shaded area?)

4. Let R be the region between the curves $y = \sin(\pi x)$ and $y = \cos(\pi x)$ from $x = 0$ to $x = \frac{1}{2}$.

(a) **[2 marks]** Let S be the solid obtained by rotating R about the x-axis. Write down, but *do not evaluate*, an expression describing the volume of S using vertical slices.

(b) **[3 marks]** Let S be the solid obtained by rotating R about the line $x = -\frac{1}{4}$. Write down, but *do not evaluate*, an expression describing the volume of S using cylindrical shells.

5. [**4 marks**] Let $B < T$ be positive constants. Let R_1 be the region enclosed by the y-axis, $y = \frac{1}{\log(x)}$, $y = B$ and $y = T$. Let C_1 be a water-filled container whose interior is exactly the same shape as the solid obtained by rotating R_1 about the y-axis.

   Let R_2 be the region enclosed by $y = \frac{1}{\log(x)}$, $y = \frac{1}{\log\left(\frac{x}{2}\right)}$, $y = B$ and $y = T$. Let C_2 be a water-filled container whose interior is exactly the same shape as the solid obtained by rotating R_2 about the y-axis.

   Prove that it takes more work to pump all of the water out the top of C_2 than to pump all the water out the top of C_1.

6. [**5 marks**] Let $f(x) = \int_x^2 \frac{1}{\sqrt{1+t^3}}\, dt$. Evaluate $\int_0^2 x f(x)\, dx$.

7. [**1 bonus mark**] Draw a short comic strip explaining how you learn mathematics.