MATH 101 V01 – ASSIGNMENT 1

There is no WeBWorK part to this first assignment. The assignment consists only of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in ***before your recitation on Friday, January 12****.*

1. Consider $\displaystyle\int_0^2 (1-2t)\,dt$.
   (a) Calculate the Riemann sum for this integral using left endpoints and 4 subintervals.
   (b) Calculate the Riemann sum for this integral using right endpoints and 4 subintervals.
   (c) Explain why, if any function f is continuous on $[l,r]$, then $\int_l^r f(t)\,dt$ may be calculated by the definition of the integral as the limit of Riemann sums using *any* choice of sample points t_i^* in each subinterval $[t_{i-1},t_i]$ of the partition.
   (d) Find the value of the integral, using the definition of the integral as the limit of Riemann sums using right endpoints $t_i^* = t_i$ in every subinterval.
   (e) Interpret your result for part (d) in terms of the areas between the curve and the horizontal axis, between $t=0$ and $t=2$.

2. Prove both the following statements, using the definition of the integral.
   (a) If f and g are integrable on $[l,r]$, then $f+g$ is integrable on $[l,r]$, and
   $$\int_l^r [f(t)+g(t)]\,dt = \int_l^r f(t)\,dt + \int_l^r g(t)\,dt,$$
   (b) If f is integrable on $[l,r]$, then for any constant c, the function cf is integrable on $[l,r]$, and
   $$\int_l^r cf(t)\,dt = c\int_l^r f(t)\,dt.$$

3. Let
   $$f(t) = \begin{cases} 1 & \text{if } 1 \le t \le \sqrt{2}, \\ 2 & \text{if } \sqrt{2} < t \le 2, \end{cases}$$
   and note that this function is not continuous on $[1,2]$.
   (a) Prove that f is integrable on $[1,2]$, and calculate $\int_1^2 f(t)\,dt$ using the definition of the integral: let n be a positive integer and use a regular partition $1 = t_0 < t_1 < t_2 < \cdots < t_{n-1} < t_n = 2$ of $[1,2]$ into n subintervals of equal width $\Delta t = (2-1)/n = 1/n$, choose sample points $t_i^* \in [t_{i-1},t_i]$ in each subinterval $i = 1,\ldots,n$, and prove that the limit $\lim_{n\to\infty}\sum_{i=1}^n f(t_i^*)\,\Delta t$ exists and is equal for all choices of sample points.
   (b) Explain in a sentence or two how some other function f on $[1,2]$ (not the function in part (a)) could be continuous everywhere except at a single point $\sqrt{2}$ in $[1,2]$, and the limit $\lim_{n\to\infty}\sum_{i=1}^n f(t_i^*)\,\Delta t$ does not exist, i.e. f is not integrable on $[1,2]$.