# MATH 101 V01 – ASSIGNMENT 3

*There are two parts to this assignment. The first part is on WeBWorK — link to it using Canvas, and go to MATH 101 V01 (after 9:00 am Fri Jan 19). The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.*

*Your written assignment must be handed in* ***before your recitation on Friday, January 26****. The online assignment will close at* ***9:00 a.m. on Friday, January 26****.*

1. (a) Let $a > 0$. Use properties of the integral and integration by substitution to prove for any function $f$ that is continuous on the closed interval $[-a, a]$:
   i. If $f$ is an *even* function ($f(-x) = f(x)$ for all $x$), then
   $$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx.$$
   ii. If $f$ is an *odd* function ($f(-x) = -f(x)$ for all $x$), then
   $$\int_{-a}^{a} f(x)\,dx = 0.$$

   (b) Let $L > 0$ be a constant and let $n$ be a positive integer. Evaluate the two integrals
   $$a_n = \frac{1}{L}\int_{-L}^{L} x^2 \cos\left(\frac{n\pi x}{L}\right)\,dx, \qquad b_n = \frac{1}{L}\int_{-L}^{L} x^2 \sin\left(\frac{n\pi x}{L}\right)\,dx.$$

2. (a) Use integration by parts to derive the following *reduction formula*
   $$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx,$$
   where $n \geq 2$ is an integer.

   (b) Use the reduction formula to evaluate
   $$\int_{0}^{\pi/2} \cos^4 x\,dx.$$