MATH 101 V01 – ASSIGNMENT 4

There are two parts to this assignment. The first part is on WeBWorK — link to it using Canvas, and go to MATH 101.V01 (after 9:00 am Friday, January 26). The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in ***before your recitation on Friday, February 2****. The online assignment will close at* ***9:00 a.m. on Friday, February 2****.*

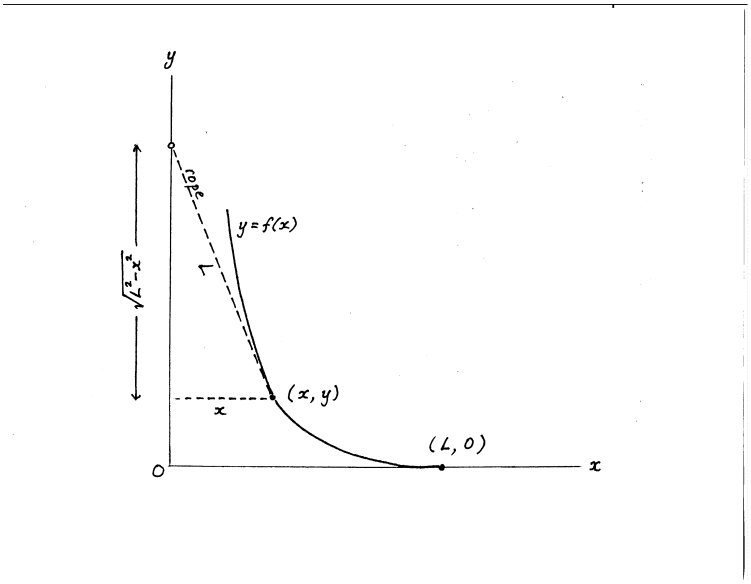
1. (a) Make the substitution $u = \sqrt{x+3}$ and then find the antiderivative (or indefinite integral)

$$\int \frac{1}{x + 2\sqrt{x+3}}\, dx.$$

   (b) Make the substitution $u = \sqrt{x-10}$ and then find the antiderivative (or indefinite integral)

$$\int \frac{1}{x + 2\sqrt{x-10}}\, dx.$$

2. Suppose the open first quadrant of the xy-plane (i.e. $x > 0$, $y > 0$) represents water, viewed from above. The rest of the xy-plane represents solid land. You are initially at the origin, holding the end of a rope of length $L > 0$ which is tied to a boat at the point $(L, 0)$ in the xy-plane. Then you walk along the positive y-axis, keeping the rope straight and taut (assume you are strong enough to pull the boat, the rope doesn't stretch, and you don't fall into the water trying to balance on the y-axis). The boat follows a path $y = f(x)$, with the property that the rope is always tangent to the path of the boat (figure below).



   (a) Show that

$$f'(x) = \frac{dy}{dx} = -\frac{\sqrt{L^2 - x^2}}{x}.$$

   (b) Determine the function $y = f(x)$.