MATH 101 V01 – ASSIGNMENT 7

There are two parts to this assignment. The first part is on WeBWorK — link to it using Canvas, and go to MATH 101.V01 (after 9:00 am Friday, March 9). The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in ***before your recitation on Friday, March 16****. The online assignment will close at* ***9:00 a.m. on Friday, March 16****.*

1. (a) Use linear approximation to estimate $\log(0.98)$.
   (b) Find the degree 2 Taylor polynomial $T_2(x)$, of the function $f(x) = x^{5/2}$, about $x = 4$.
   (c) Find the degree 3 Taylor polynomial $T_3(x)$, of the function $f(x) = \sqrt{x}$, about $x = 4$.
   (d) Find the degree 5 Taylor polynomial $T_5(x)$, of the function $f(x) = \cos(x)$, about $x = \frac{\pi}{3}$.
   (e) Find the degree 8 Taylor polynomial $T_8(x)$, of the function $f(x) = \cos(x)$, about $x = 0$ (a Taylor polynomial about $x = 0$ is called a **Maclaurin polynomial**).

2. (a) Find an upper bound on the absolute value of the error made if linear approximation about $x = 4$ is used to estimate $(3.9)^{5/2}$, and determine (without calculating the "exact" value numerically) whether this approximation is greater than, or less than, the exact value $(3.9)^{5/2}$.
   (b) Find an upper bound on the absolute value of the error made if the degree 2 Taylor polynomial about $x = 4$ is used to estimate $\sqrt{4.2}$, and determine (without calculating the "exact" value numerically) whether this approximation is greater than, or less than, the exact value $\sqrt{4.2}$.
   (c) Determine what degree n of Taylor polynomial $T_n(x)$, of the function $f(x) = \cos(x)$, about $x = \frac{\pi}{3}$ is needed to guarantee that the Taylor polynomial approximation of $\cos(69°)$ is accurate within 5×10^{-6} (i.e. the error is guaranteed to have an absolute value no larger than 5×10^{-6}).
   (d) Determine what degree n of Maclaurin polynomial $T_n(x)$, of the function $f(x) = \log(1+x)$, is needed to guarantee that the Maclaurin polynomial approximation of $\log(1.4)$ is accurate within 10^{-3}.

3. Let R be the region between the y-axis and the curve $x = (16 + y)^{1/4}$, with $-16 \le y \le 0$, and both x and y measured in metres. The region R is rotated around the y-axis, creating a volume. This volume is filled with a fluid that has volume density 888 kg/m^3. Determine the work done (in joules) pumping all the fluid up to $y = 0$. Use $g = 9.8$ m/s^2 for the acceleration due to gravity. (You must evaluate the integral and a calculator-ready answer is sufficient.)