MATH 101 V01 – ASSIGNMENT 8

There are two parts to this assignment. The first part is on WeBWorK — link to it using Canvas, and go to MATH 101.V01 (after 9:00 am Friday, March 16). The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in ***before your recitation on Friday, March 23****. The online assignment will close at* ***9:00 a.m. on Friday, March 23****.*

1. (a) Find a power series representation for $f(x) = x\sin(x/2)$ and determine the interval of convergence.
   (b) Find the first four nonvanishing terms in the alternating series representation of $\int_0^{1/2} \arctan(x^3)\,dx$.
   (c) Evaluate $\lim_{x\to 0} \frac{-x+\sin(x)}{x^4}$, or determine that the limit does not exist.
   (d) Evaluate $\lim_{x\to 0} \frac{x^2-2+2\cos(x)}{x^4}$, or determine that the limit does not exist.
   (e) If $f(x) = 2\sin(x)\cos(x)$, find $f^{(101)}(0)$.

2. Let $f(x) = (1+x)^\alpha$, where α is any fixed real number.
   (a) Find the Maclaurin series of $(1+x)^\alpha$.
   (b) Find the radius of convergence of the Maclaurin series.
   (c) The length L of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b > 0$, is (you do not have to show this)

$$L = 4a \int_0^{\pi/2} \sqrt{1 - \epsilon^2 \sin^2(\theta)}\, d\theta,$$

   where $\epsilon = \frac{\sqrt{a^2-b^2}}{a}$ is the *eccentricity* of the ellipse. If ϵ is near 0, the ellipse is nearly a circle. Use part (a) to find the first three nonvanishing terms in the series representation of L, in powers of ϵ. Use the series to estimate the length of the ellipse with $a = 1.01$, $b = 0.99$.

3. (a) Define, using Riemann sums, what it means for a function $f(x)$ to be integrable on a closed interval $[l, r]$, where $l < r$.
   (b) Let

$$f(x) = \begin{cases} 1 & \text{if } x = j/2^k \text{ for integers } j \text{ and } k, \text{ with } k \text{ positive and } 0 \le j \le 2^k, \\ -1 & \text{otherwise.} \end{cases}$$

   Prove that $f(x)$ is not integrable on $[0, 1]$.