MATH 215 Section 202

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2 Websites

i. Canvas site canvas.ubc.ca/courses/37708

ii. Instructor site
www.math.ubc.ca/~nagata/m215/

Textbook: online (free download pdf)
see Canvas page.

Read 0.2, 0.3

1. First order ODEs

Read 1.1

Some definitions:

Differential Equation (DE) eqn. for an unknown function, that contains derivatives of that function e.g. $y' = xy$

Ordinary DE (ODE): unknown function depends on one independent variable (typically x or t)

Order of a DE: highest derivative that appears in the DE.

e.g. $y' = xy$ is 1st order

e.g. $y'' + yy' + \sin(y) = A\cos(\omega t)$ is a 2nd order ODE

Solution of a DE: a "continuous" function that satisfies the DE when it is substituted into the equation.

Exercise Verify that

$$y(t) = C_1\cos(2t) + C_2\sin(2t)$$

is a solution of

$$y'' + 4y = 0, \quad -\infty < t < +\infty$$

for arbitrary constants C_1, C_2.

Initial condition (IC): value(s) of the function (and derivative(s)) specified at some given value of the indep. variable (e.g. at "time" $t = 0$)

Initial value problem (IVP): DE + IC

Exercise Find "the" solution of the IVP

$$\begin{cases} y'' + 4y = 0 \\ y(0) = 0, \quad y'(0) = 1 \end{cases}$$

1.2 Slope fields

General 1st order ODE

$$\frac{dy}{dx} = f(x,y)$$

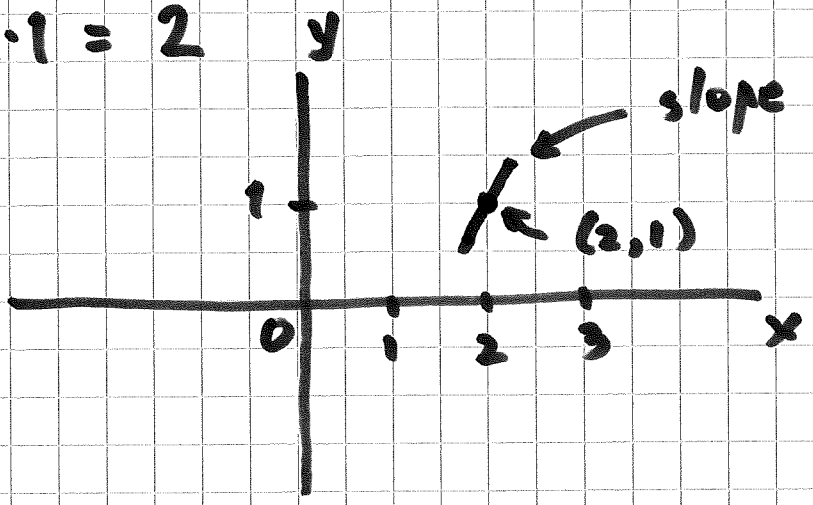
1.2.1 Slope fields

$\frac{dy}{dx}$ = slope of tangent line to curve $y(x)$ is specified as $f(x,y)$ at (x,y)

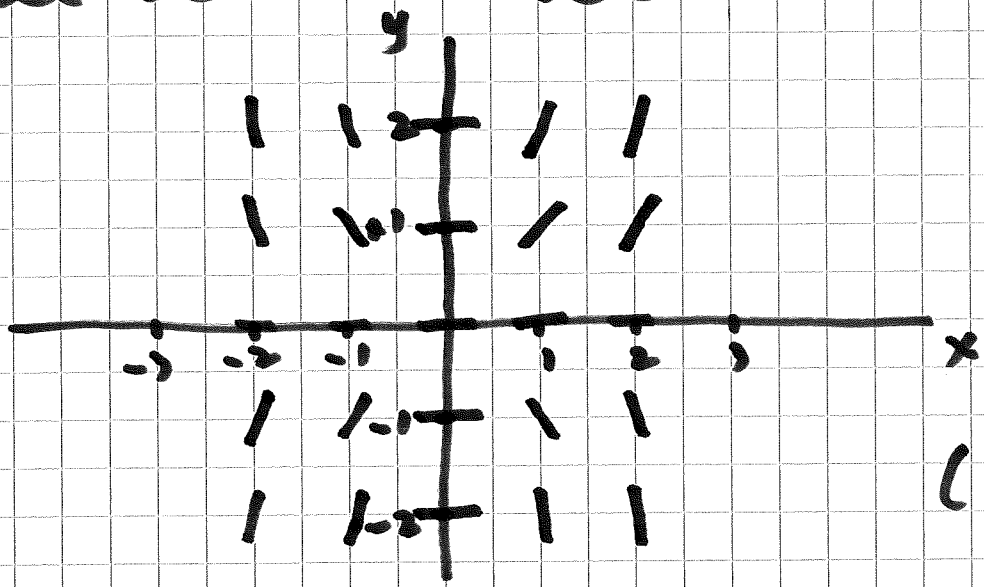
Example 1.A $\frac{dy}{dx} = xy$

At the point $(x,y) = (2,1)$ the slope is

$xy = 2 \cdot 1 = 2$



Do this for many points (by hand, or by Matlab), get a picture of the slope field (or direction field)



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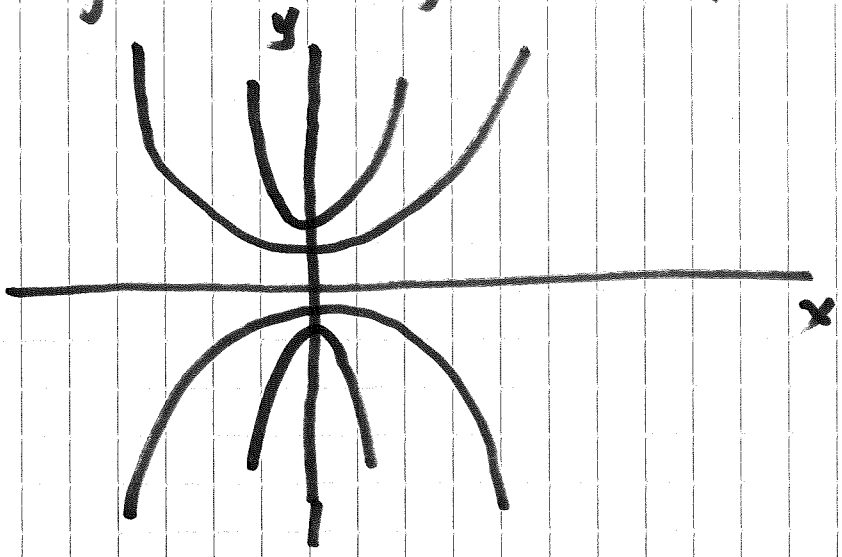
Graphs of solutions (called solution curves) must always be tangent to line segments in the slope field

We can guess using the slope field



1.2.2 Existence and uniqueness

Theorem 1.2.1 (Picard's theorem) If $f(x,y)$ and $\frac{\partial f}{\partial y}(x,y)$ are both continuous functions of (x,y) in a neighbourhood of a point (x_0, y_0), then the IVP

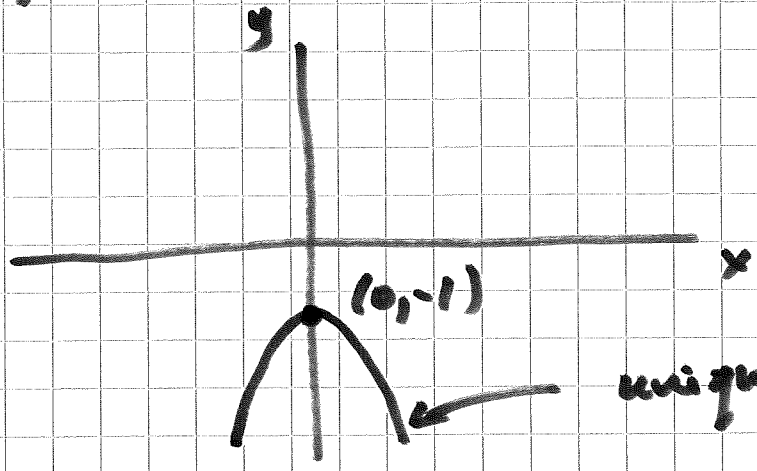
$$\frac{dy}{dx} = f(x,y),$$
$$y(x_0) = y_0$$

has a unique solution that exists for some open interval of x-values that contains x_0.

Typically there are many solutions of an ODE $\frac{dy}{dx} = f(x,y)$

but only one solution of an IVP

e.g.



unique sol. of

$\frac{dy}{dx} = xy$

$y(0) = -1$

1.3 Separable equations

A separable 1st order ODE is a 1st order ODE that can be written in the form

$$\frac{dy}{dx} = f(x)\,g(y)$$

Solve by "separating variables" and integrating:

$$\frac{1}{g(y)}\,dy = f(x)\,dx$$

$$\int \frac{1}{g(y)}\,dy = \int f(x)\,dx + C$$