HW 1: Fri Jan 24 11:59pm

If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M}$ is a <u>function of y only</u>

then

$$\frac{du}{dy} = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} u$$

and $u(y)$ satisfies a linear ODE, can solve.

<u>Example</u> 1.8.B cont.

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{\frac{2y}{x+1} - 0}{2y}$$

$$= \frac{1}{x+1}$$

is a function of x only: look for integrating factor $u = u(x)$ only.

Then $u(x)$ satisfies

$$\frac{du}{dx} = \frac{1}{x+1} u$$

Solve (exercise : separable, linear)

$$u(x) = x + 1$$

(Find <u>any</u> nonzero integrating factor, don't need the most general one)

Multiply the original ODE by $x+1$:

$$x^2+y^2 + 2y(x+1)\frac{dy}{dx} = 0$$

This exact, can be solved (see Ex. 1.8.A)

Read Ex. 1.8.7 (p. 69).

1.7 <u>Numerical methods: Euler's method</u>

$$\frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0$$

Approximate the solution : use the slope field.

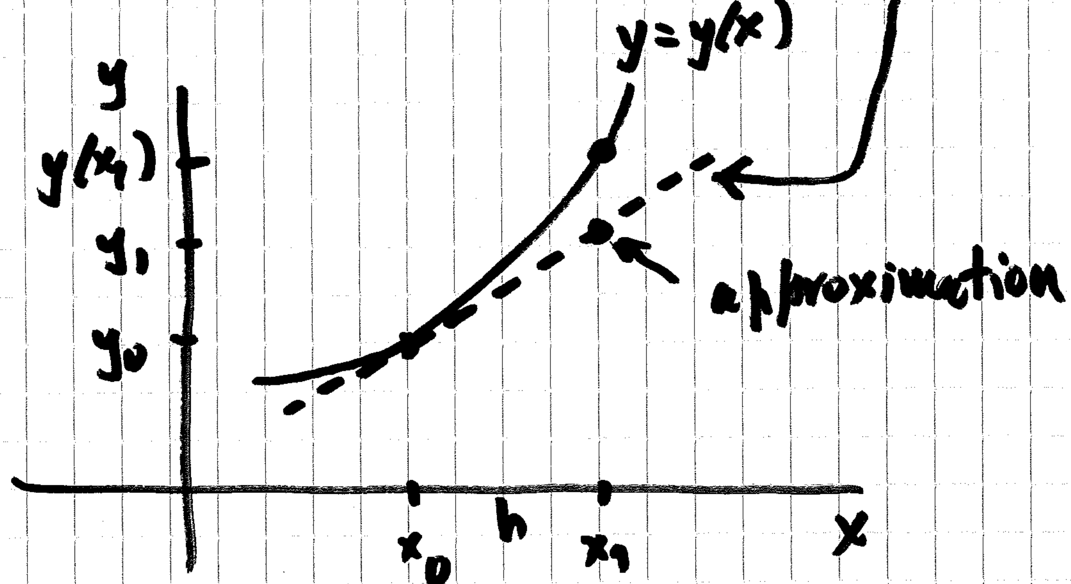
We know (x_0, y_0) is on the graph of $y(x)$.

At this point (x_0, y_0), the slope of the tangent line to $y(x)$ is

$$\frac{dy}{dx}(x_0) = f(x_0, y_0)$$

For nearby $x_1 = x_0 + h$, $h > 0$ small we approximate $y(x)$ by its tangent line

$$y = y_0 + \overbrace{f(x_0, y_0)}^{\text{slope}}(x - x_0)$$



At $x = x_1 = x_0 + h$:

$$y(x_1) \approx y_1 = y_0 + h\, f(x_0, y_0)$$

Continue making approximations by updating with the formula

$$\boxed{x_{i+1} = x_i + h \quad , \quad y_{i+1} = y_i + h f(x_i, y_i)}$$

$i = 0, 1, 2, \ldots$

Euler's method to approximate solu. of ODE

Example 1.7. A

Use Euler's method with step size

(a) 0.1 (b) 0.05

to approximate $y(0.2)$ where $y(x)$ is the solution of

$$\frac{dy}{dt} = 4y + 1 - t \quad , \quad y(0) = 1 .$$

(MATLAB orientation : 20 min)