HW 1: Fri 11:59 pm

(7) Wed 2020-01-22

Office hr moved Fri Jan 24 only
to 12 noon - 12:50pm

(a) $h = 0.1$, $n = \frac{0.2 - 0}{h} = 2$ steps needed

(i=0) $t_0 = 0$ $\quad y_0 = 1$

(i=1) $t_1 = 0 + 0.1 = 0.1$ $\quad y_1 = 1 + 0.1(4 * \underbrace{1}_{y_0} + 1 - \underbrace{0}_{t_0}) = 1.5$

(i=2) $t_2 = 0.1 + 0.1 = 0.2$ $\quad y_2 = 1.5 + 0.1(4 * 1.5 + 1 - 0.1) = 2.19 \quad (= y_2[0.1])$

$y(0.2) \approx 2.19$

(b) $h = 0.05$, $n = \frac{0.2 - 0}{h} = 4$ steps

(i=0) $t_0 = 0$ $\quad y_0 = 1$

(i=1) $t_1 = 0.05$ $\quad y_1 = 1.25$

(i=2) $t_2 = 0.1$ $\quad y_2 = 1.5475$

(i=3) $t_3 = 0.15$ $\quad y_3 = 1.9020$

(i=4) $t_4 = 0.2$ $\quad y_4 = 2.3249 = y_4[0.05]$

$y(0.2) \approx 2.3249$

Which do you think is more accurate?

Verify by calculating $|\text{Error}|$ (abs. value)

Exercise Solu. of IVP

is $y(t) = \frac{1}{4}t - \frac{3}{16} + \frac{19}{16}e^{4t}$

$y(0.2) = 2.5053$

$$\boxed{\text{Error} = \text{Exact} - \text{Approximation}}$$

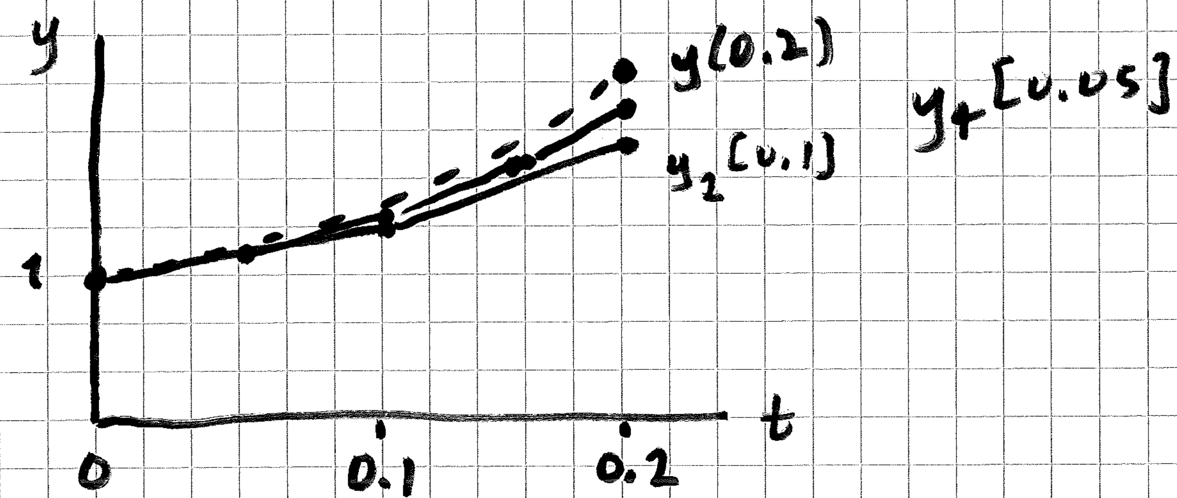
Error after $n=2$ steps of size $h=0.1$ is

$$E_2[0.1] = y(0.2) - y_2[0.1]$$
$$= 2.5053 - 2.19 = 0.3153$$

Error after $n=4$ steps of size $h=0.05$ is

$$E_4[0.05] = y(0.2) - y_4[0.05]$$
$$= 2.5053 - 2.3249 = 0.1814$$

$|E_4[0.05]|$ is smaller, $y_4[0.05]$ is more accurate



2. Higher order linear ODEs

2.1 Second order linear ODEs

Can be written in form

$$A(x)y'' + B(x)y' + C(x)y = F(x)$$

Linear in y, y', y''.

A, B, C are "coefficient (functions)"

Divide by $A(x) \neq 0$ to get "standard form"

$$(2.1) \qquad y'' + p(x)y' + q(x)y = f(x)$$

~~Theorem~~ Homogeneous if $f(x) \equiv 0$

Theorem 2.1.1 (Superposition) If $y_1(x), y_2(x)$ are two solutions of the homog. equ.

$$(2.2) \qquad y'' + p(x)y' + q(x)y = 0$$

then so is

$$y(x) = C_1 y_1(x) + C_2 y_2(x)$$

for arbitrary constants C_1, C_2

Proof: Exercise

<u>Operator notation</u>: the operator L is

$$L = \frac{d^2}{dx^2} + p(x)\frac{d}{dx} + q(x) \quad ,$$

$$Ly = y'' + p(x)\,y' + q(x)\,y$$

so (2.1) becomes $\quad Ly = f(x)$

(2.2) becomes $\quad Ly = 0$

<u>Theorem</u> 2.1.2 (Existence-uniqueness)
If $p(x)$, $q(x)$, $f(x)$ are all continuous for x in an open interval I, and if a, b_0, b_1 are given constants with $a \in I$, then the IVP

$$Ly = f(x) \quad , \quad y(a) = b_0 \quad , \quad y'(a) = b_1$$

has a solution $y(x)$, it is unique, and it is defined in I.

Two <u>functions</u> $y_1(x)$, $y_2(x)$ are <u>linearly independent</u> in an interval I if

$$C_1\,y_1(x) + C_2\,y_2(x) \equiv 0 \quad \Rightarrow \quad C_1 = 0 \;\&\; C_2 = 0$$

Easier criterion: two <u>solutions</u> $y_1(x)$, $y_2(x)$ of the <u>homogeneous equ.</u> $Ly = 0$ are lin. indep. if and only if one solution is not a constant multiple of the other solution.