Homework HW2 Feb 14 (useful for MT1) 11 Fri 2020-01-31

Midterm MT1 Feb 7

See Canvas

Webwork W2 11:59 pm

$$x(t) = -0.02\cos(20t)$$

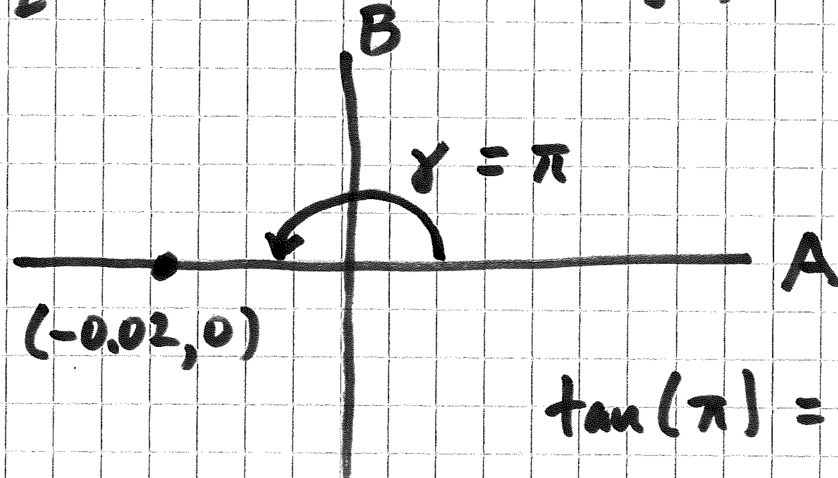
$$(\omega_0 = 20,\ A = -0.02,\ B = 0)$$

Amplitude: $C = \sqrt{A^2 + B^2} = \sqrt{(-0.02)^2 + (0)^2}$

$= 0.02$

Phase shift: $\tan(\gamma) = \dfrac{B}{A} = 0 \quad (0 \le \gamma < 2\pi)$

Check quadrant: $A = \cos(\gamma)$, $B = \sin(\gamma)$




$\tan(\pi) = 0$ ✓

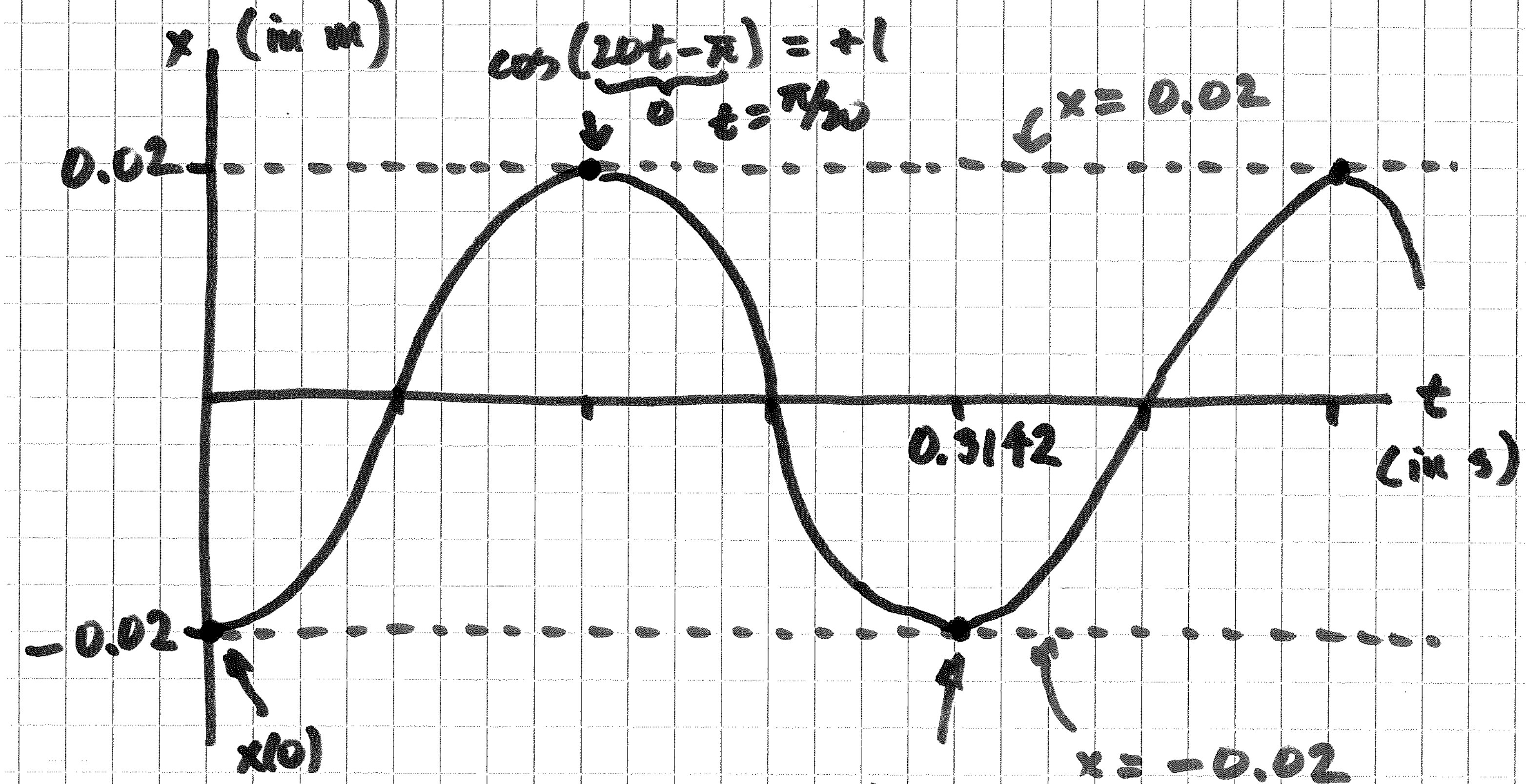
$\gamma = \pi$ (not $\arctan(0)$)

Amplitude 0.02 m

Phase shift $\pi$ (radians)

no units

$$x(t) = 0.02 \cos(20t - \pi)$$




$\cos(20t-\pi) = -1$

$t = 0$

$\cos(\underbrace{20t-\pi}_{\pi}) = -1$

$t = \frac{2\pi}{20} = 0.3142$

$\underbrace{\qquad\qquad\qquad\qquad}$

$$\text{length of one period} = \frac{2\pi}{\omega_0} = \frac{2\pi}{20} = \frac{\pi}{10}$$
$$= 0.3142$$

$F(t)\equiv 0$ $\quad c>0$

## 2.4.3 Free damped motion

$$mx'' + cx' + kx = 0$$

$m, c, k$ all positive constants

Characteristic eqn.

$$mr^2 + cr + k = 0 \qquad r_1, r_2 = \frac{-c \pm \sqrt{c^2-4mk}}{2m}$$

Case i. $c^2-4mk>0$ "overdamped" mass-spring system

$r_1, r_2$ are real, distinct, negative (why?)

$$x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

$$\lim_{t\to\infty} x(t) = 0 \qquad \text{(why?)}$$

Case ii. $c^2-4mk=0$ "critically damped"

$r_1, r_2$ are real, not distinct, negative

$$r_1 = r_2 = -\frac{c}{2m} = -p$$

where $p = \frac{c}{2m} > 0$

$$x(t) = C_1 e^{-pt} + C_2 t e^{-pt}$$

$$\lim_{t\to\infty} x(t) = 0 \qquad \text{(Exercise)}$$

Case iii. $c^2 - 4mk < 0$ "underdamped"

$r_1, r_2$ are not real, distinct, <u>negative real parts</u>

$$r_1, r_2 = -p \pm i\omega_1$$

where $\left.\begin{aligned} p &= \frac{c}{2m} > 0 \\ \omega_1 &= \frac{\sqrt{4mk - c^2}}{2m} > 0 \end{aligned}\right\}$ Exercise

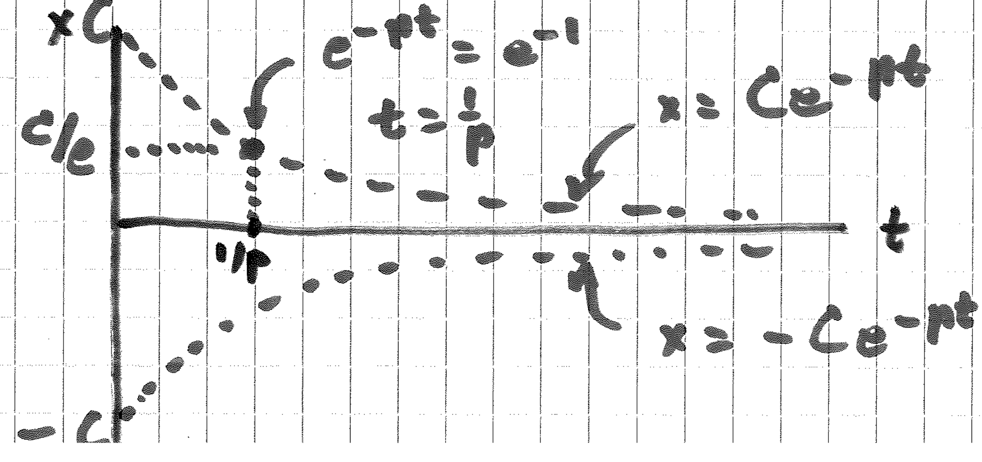
$\omega_1$: angular <u>pseudo-frequency</u>

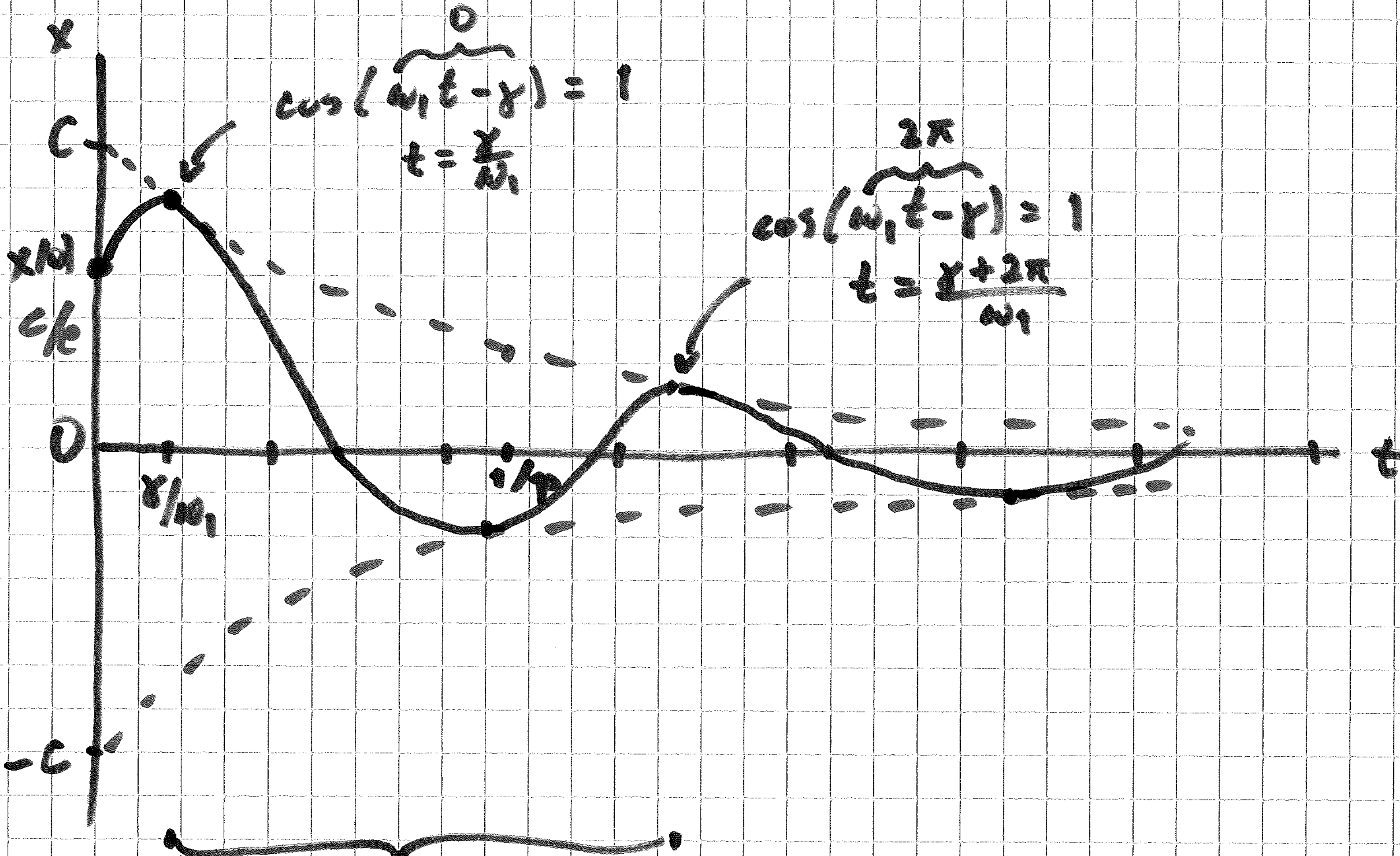
$$x(t) = A e^{-pt}\cos(\omega_1 t) + B e^{-pt}\sin(\omega_1 t)$$

$$= e^{-pt}\Big[\underbrace{A\cos(\omega_1 t) + B\sin(\omega_1 t)}_{= C\cos(\omega_1 t - \gamma)}\Big]$$

$$= C e^{-pt}\cos(\omega_1 t - \gamma) \qquad (C > 0)$$

$$-1 \le \cos(\omega_1 t - \gamma) \le 1, \quad -Ce^{-pt} \le x(t) \le Ce^{-pt}$$






length of one ~~s/s~~ pseudoperiod $= \frac{2\pi}{\omega_1}$

<u>Example</u> 2.4.B A mass of 0.02 kg hangs from a spring and stretches it 0.05 m at rest. The mass is attached to a device that resists the motion with a force of 4 N if the speed is 10 $m \cdot s^{-1}$. The mass is pulled down 0.02 m from its rest position and released with an upwards velocity of 0.2 $m \cdot s^{-1}$. Find the displacement of the mass below the rest position at any later time.

$$mx'' + cx' + kx = 0$$ (x is downwards displacement)

$m = 0.02$ kg

$$c = \frac{4\ N}{10\ m \cdot s^{-1}} = 0.4\ N \cdot s \cdot m^{-1}$$

$$k = \frac{mg}{L} = \frac{(0.02\ kg)(9.8\ m \cdot s^{-2})}{0.05\ m} = 3.92\ N \cdot m^{-1}$$

$$0.02\, x'' + 0.4\, x' + 3.92\, x = 0$$

$$\begin{cases} x'' + 20\, x' + 196\, x = 0 \\ x(0) = 0.02, \quad x'(0) = -0.2 \end{cases}$$