HW 2 Feb 14
MT 1 Feb 7

12 Mon 2020-02-03

Characteristic equ.

$$r^2 + 20r + 196 = 0$$

roots

$$r_1, r_2 = \frac{-20 \pm \sqrt{400 - 4(1)(196)}}{2} = -10 \pm i\, 4\sqrt{6}$$

$$\left(\alpha = -10, \quad \beta = 4\sqrt{6} \right.$$

General solution

$$\left. p = 10, \quad \omega_1 = 4\sqrt{6} \right)$$

$$x(t) = A e^{-10t} \cos(4\sqrt{6}\, t) + B e^{-10t} \sin(4\sqrt{6}\, t)$$

$$x'(t) = A\left[-10 e^{-10t} \cos(4\sqrt{6}\, t) - 4\sqrt{6}\, e^{-10t} \sin(4\sqrt{6}\, t)\right]$$
$$+ B\left[-10 e^{-10t} \sin(4\sqrt{6}\, t) + 4\sqrt{6}\, e^{-10t} \cos(4\sqrt{6}\, t)\right]$$

At $t = 0$

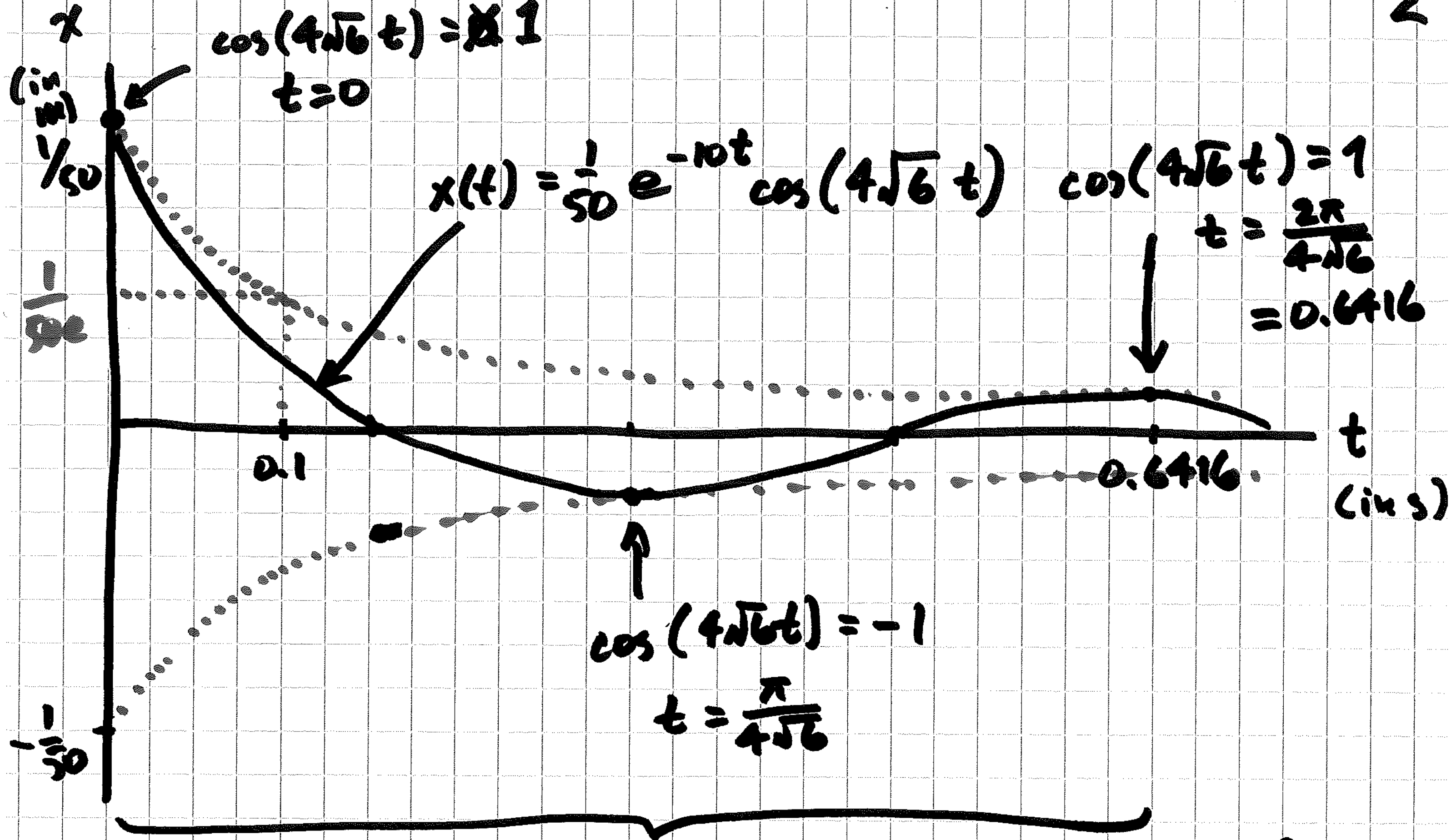
$$\left.\begin{aligned} x(0) &= A &&= 0.02 \\ x'(0) &= -10A + 4\sqrt{6}\, B &&= -0.2 \end{aligned}\right\} \Leftrightarrow \begin{aligned} A &= \tfrac{1}{50} \\ B &= 0 \end{aligned}$$

Solu. of IVP

$$x(t) = \frac{1}{50} e^{-10t} \cos(4\sqrt{6}\, t) \quad \text{m} \quad (t \text{ in s})$$

Already in form $\quad C e^{-pt} \cos(\omega_1 t - \gamma)$

$$C = \frac{1}{50} \quad \gamma = 0 \quad (p = 10, \ \omega_1 = 4\sqrt{6})$$



length of one pseudo-period $= \frac{2\pi}{\omega_1}$

$= 0.6416$

2.5 Nonhomogeneous equations

2.5.1 Solving nonhomogeneous equations

$$\underbrace{y'' + p(x)\,y' + q(x)\,y}_{Ly} = f(x)$$

<u>Theorem</u> 2.5.1 The general solution to $Ly = f(x)$ has the form

$$y(x) = y_c(x) + y_p(x)$$

where $y_c(x) = C_1 y_1(x) + C_2 y_2(x)$ is the general solu. to the <u>associated homogeneous equ.</u> $Ly = 0$, called the <u>complementary solution</u>, and $y_p(x)$ is any (specific) <u>particular solution</u> to $Ly = f(x)$.

2.5.2 <u>Undetermined coefficients</u> ("guess & check")

If Ly has <u>constant coefficients</u> and $f(x)$ is "nice" (see table below), then the form of $y_p(x)$ can be guessed. If appropriate, write $f(x) = f_1(x) + \ldots + f_m(x)$ where each $f_j(x)$ is "nice", and put $y_p(x) = y_{p_1}(x) + \ldots + y_{p_m}(x)$ and guess each $y_{p_j}(x)$.

Table

If $f_j(x)$ has the form...	... then guess* $y_{p_j}(x)$
$P_n(x) = a_0 + a_1 x + \ldots + a_n x^n$	$x^s\left[A_0 + A_1 x + \cdots + A_n x^n\right]$
$P_n(x) e^{\alpha x}$	$x^s\left(A_0 + \ldots + A_n x^n\right) e^{\alpha x}$
$P_n(x) e^{\alpha x} \begin{cases} \cdot \cos(\beta t) \\ \cdot \sin(\beta t) \end{cases}$	$x^s\left[\left(A_0 + \ldots + A_n x^n\right) e^{\alpha x} \cos(\beta t) + \left(B_0 + \ldots + B_n x^n\right) e^{\alpha x} \sin(\beta t)\right]$

* with $s = 0$ or 1 or 2 the <u>least</u> value that ensures <u>no</u> term in $y_{p_j}(x)$ is a solution of $Ly = 0$

<u>Example</u> 2.5.A Solve
$$y'' - y' + y = e^{2x}$$
$$y(0) = \frac{2}{3}, \quad y'(0) = \frac{5}{6}$$

(Associated homogeneous equ. is
$$y'' - y' + y = 0$$
)

Characteristic equ.
$$r^2 - r + 1 = 0$$

Roots
$$r_1, r_2 = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm i \frac{\sqrt{3}}{2}$$

Complementary soln.

$$y_c(x) = C_1 e^{\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 e^{\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$$

(C_1, C_2 arbitrary constants)

Now find $y_p(x)$. $\quad f(x) = e^{2x}$

Guess $\quad y_p(x) = x^s A e^{2x} \qquad s = 0$ or 1 or 2

Try $s=0$ <u>first</u>: is $x^0 A e^{2x} = A e^{2x}$ a solution of $y'' - y' + y = 0$? <u>No</u>! (Why?)

So try $\quad y_p(x) = A e^{2x}$:

$$y_p''(x) - y_p'(x) + y_p(x) \stackrel{?}{=} e^{2x}$$

$$(Ae^{2x})'' - (Ae^{2x})' + Ae^{2x} \stackrel{?}{=} e^{2x}$$

$$4Ae^{2x} - 2Ae^{2x} + Ae^{2x} \stackrel{?}{=} e^{2x}$$

$$(4A - 2A + A)e^{2x} \stackrel{?}{=} e^{2x}$$

Yes if $3A = 1$.

$$y_p(x) = \frac{1}{3} e^{2x}$$