HW4 Fri

25 Wed 2020-03-11

3 Table 8.1

	Eigenvalues	(Local) behaviour	Stability
1	real distinct pos.	(nodal) source/ unstable node	unstable
2	real distinct neg.	(nodal) sink/ stable node	asymptotically stable
3	real and opposite signs	saddle	unstable
4	complex with positive real parts	spiral source	unstable
5	complex with negative real parts	spiral sink	asymptotically stable
6	real double pos.	?	unstable
7	real double neg.	?	asymptotically stable

In the first five cases, the local phase portrait (i.e. $(x(t), y(t))$ near (x_0, y_0)) is a slightly distorted version of the phase portrait of the linearization for $(u(t), v(t))$ near $(0,0)$.

Farther away from (x_0, y_0) we expect the linearization to be less useful.

Example 8.1.A cont. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} y \\ -y-x+x^2 \end{bmatrix}$

(3) Use linearization to classify the local behaviour (if possible) and find the stability (if possible) at each critical point.

(a) At $(x_0, y_0) = (0,0)$, $\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$

$$0 = \begin{vmatrix} 0-\lambda & 1 \\ -1 & -1-\lambda \end{vmatrix} = \lambda^2 + \lambda + 1$$

$$\lambda_{1,2} = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

Eigenvalues

$$\lambda_1 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \quad \lambda_2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$

are complex with negative real parts

Local behaviour at $(0,0)$ is a spiral sink, $(0,0)$ is asymptotically stable.

(b) At $(x_0, y_0) = (1,0)$, $\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$

$$0 = \begin{vmatrix} 0-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = \lambda^2 + \lambda - 1$$

$$\lambda_{1,2} = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

Eigenvalues

$$\lambda_1 = \frac{-1-\sqrt{5}}{2} < 0 \,, \quad \lambda_2 = \frac{-1+\sqrt{5}}{2} > 0$$

are real and opposite signs.

Linear behaviour at $(1,0)$ is a saddle,

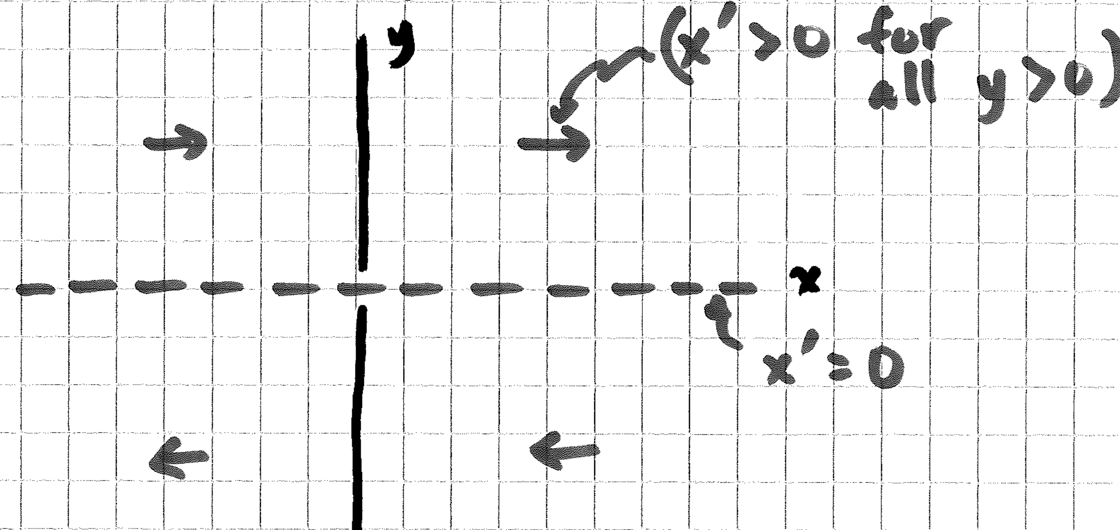
$(1,0)$ is unstable.

4 Draw nullclines (and direction field) and local phase portraits (near each crit. pt.)

$x' = y$ $\qquad$ $x' > 0 \Leftrightarrow y > 0$ (upper half plane)

$\qquad\qquad\quad x' = 0 \Leftrightarrow y = 0$

$\qquad\qquad\quad x' < 0 \Leftrightarrow y < 0$



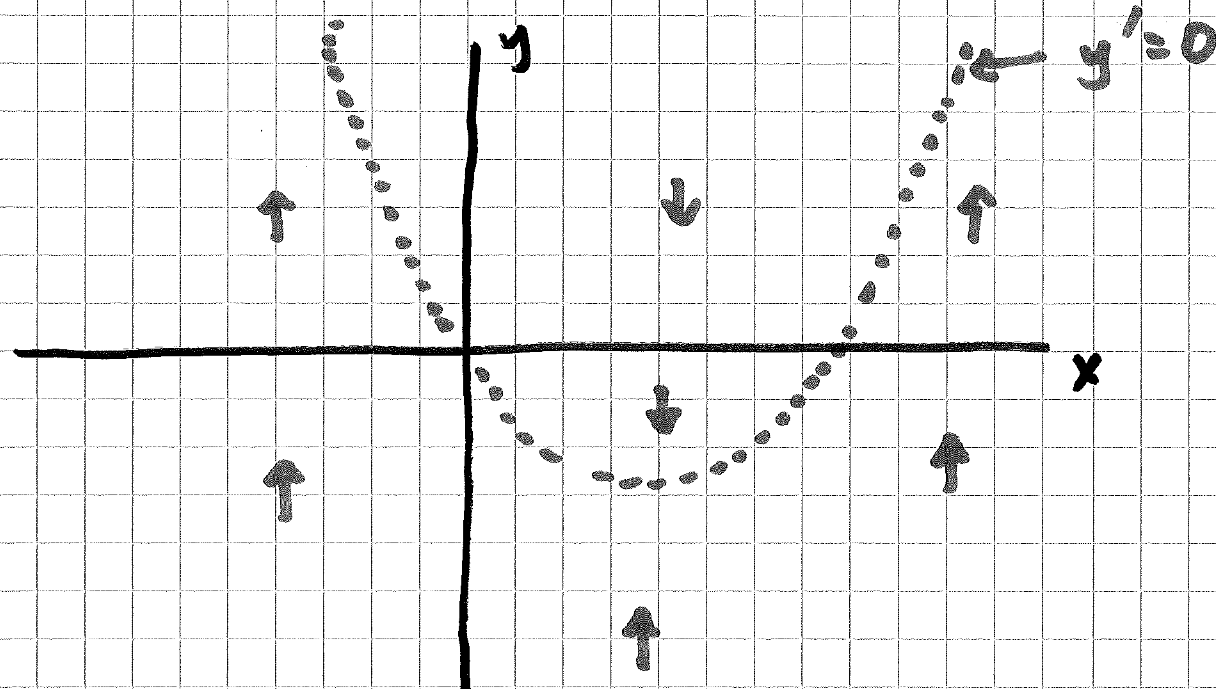
$$y' = -y - x + x^2$$

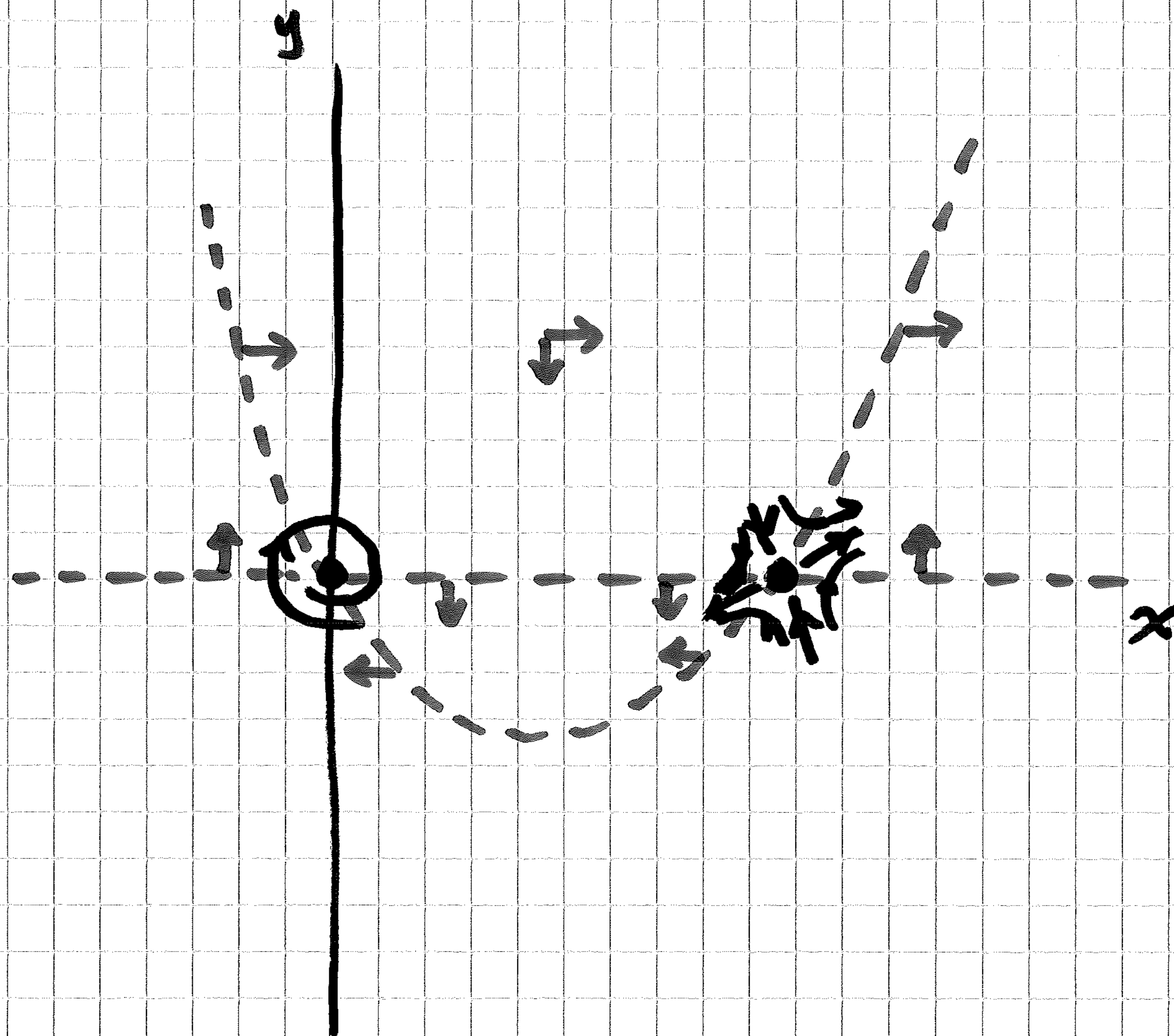
$$y' > 0 \Leftrightarrow -y - x + x^2 > 0$$

$$y < -x + x^2 = x(x-1)$$

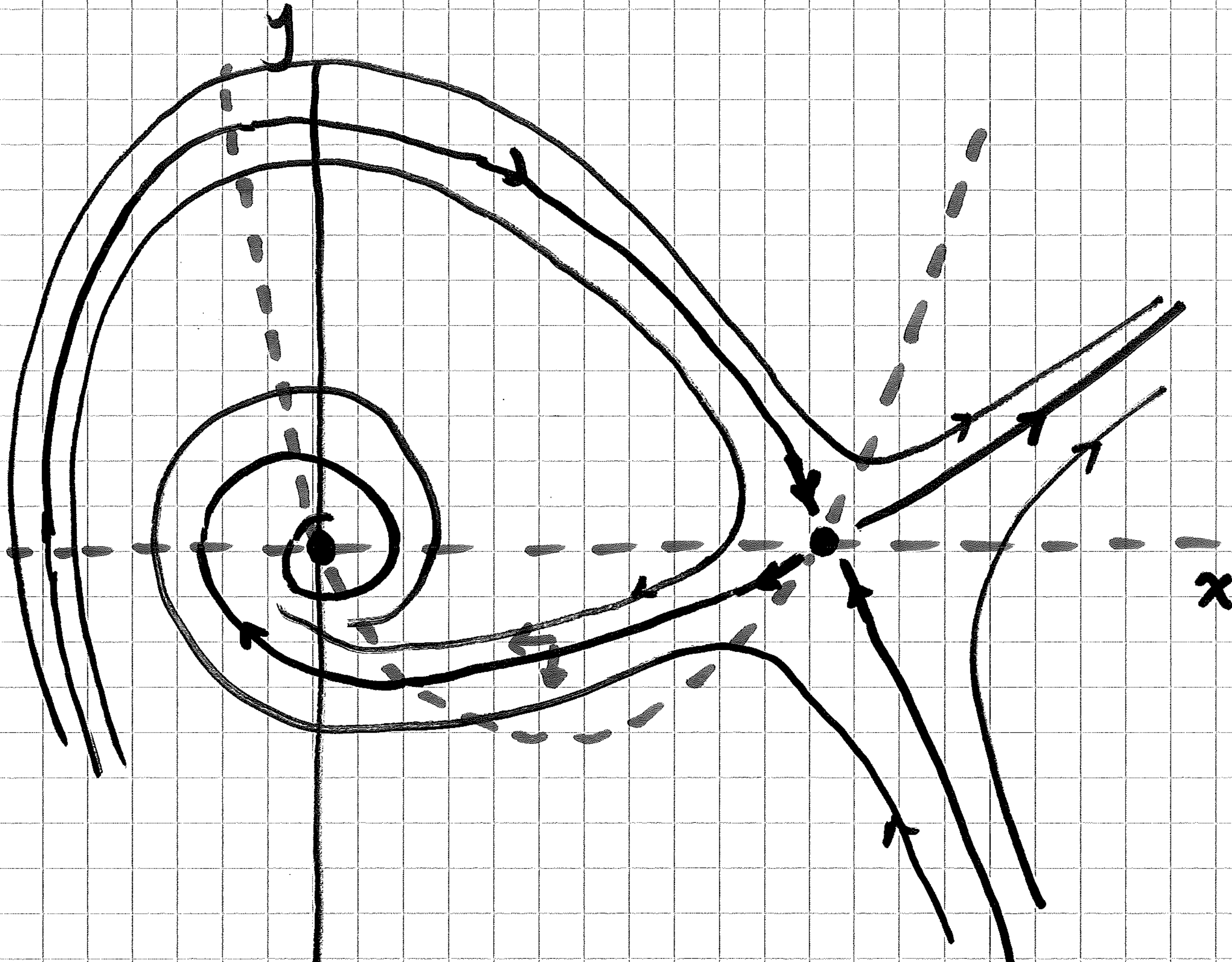
$$y' = 0 \Leftrightarrow y = -x + x^2$$

$$y' < 0 \Leftrightarrow y > -x + x^2$$





(5) Draw the (global) phase portrait
(consistent with above analysis)


y
x