Chapter 6 The Laplace transform

6.1 The Laplace transform

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Definition: for a (piecewise continuous) function $f(t)$, $t \geq 0$ we define its Laplace transform to be

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

(provided the improper integral converges).

Example 6.1.A. Use the definition to find $\mathcal{L}\{e^{-at}\}$, where a is a constant.

$$F(s) = \mathcal{L}\{e^{-at}\} = \int_0^\infty e^{-st}\, e^{-at}\, dt$$

$$= \int_0^\infty e^{-(s+a)t}\, dt$$

$$= -\frac{1}{s+a}\, e^{-(s+a)t}\Big|_0^\infty$$

$$= \underbrace{\lim_{b\to\infty}\left[-\frac{1}{s+a}\, e^{-(s+a)b}\right]}_{=0 \text{ if } s+a>0} - \left[-\frac{1}{s+a}\, \underbrace{e^{-(s+a)0}}_{=1}\right]$$

$$= \frac{1}{s+a}$$

$$\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$$ (provided $s > -a$ so that the improper integral converges)