To help find other inverse Laplace transforms, we have:

Theorem (First shifting property; "s-shift")

$$\mathcal{L}\left\{ e^{-at} f(t) \right\} = F(s+a),$$

where $\mathcal{L}\{f(t)\} = F(s)$. Equivalently,

$$\mathcal{L}^{-1}\left\{ F(s+a) \right\} = e^{-at} f(t).$$

Exercise. Prove it.

e.g. if $f(t)=1$ then $F(s)=\mathcal{L}\{1\}=\frac{1}{s}$, and $\mathcal{L}\ \{e^{-at}\cdot 1\}=\frac{1}{s+a}$

by the 1st shifting property, and also consistent with Table 6.1.

Typical use of 1st shifting property:

Example 6.1.D. Find $\mathcal{L}^{-1}\left\{\frac{s+5}{s^2+2s+5}\right\}$

(Looks similar to Ex. 6.1.C, but requires different technique!)

Does s^2+2s+5 factor (over the reals)? **No**.

This time, we **complete the square**:

$$s^2+2s+5 = s^2+2s+1+4 = (s+1)^2+4$$

(Observe that $\frac{s}{s^2+\omega^2}$, $\frac{\omega}{s^2+\omega^2}$ are in Table 6.1; use $\omega=2$ and shift s to $s+1$)

Then,

$$\frac{s+5}{s^2+2s+5} = \frac{s+5}{(s+1)^2+4}$$

$$= \frac{s+1+4}{(s+1)^2+2^2}$$

$$= \frac{s+1}{(s+1)^2+2^2} + 2\,\frac{2}{(s+1)^2+2^2}$$

so we have

$$\mathcal{L}^{-1}\left\{\frac{s+5}{s^2+2s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+2^2}\right\} + 2\,\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2+2^2}\right\}$$

$$= e^{-t}\cos(2t) + 2\,e^{-t}\sin(2t)$$

where we used $a=1$ in the first shifting property,

and Table 6.1:

$$\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2+\omega^2}$$

$$\mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2+\omega^2}$$

with $\omega = 2$.