Bessel functions

Bessel's ODE: $$\frac{d^2y}{dz^2} + \frac{1}{z}\frac{dy}{dz} + y - \frac{m^2}{z^2}y = 0$$

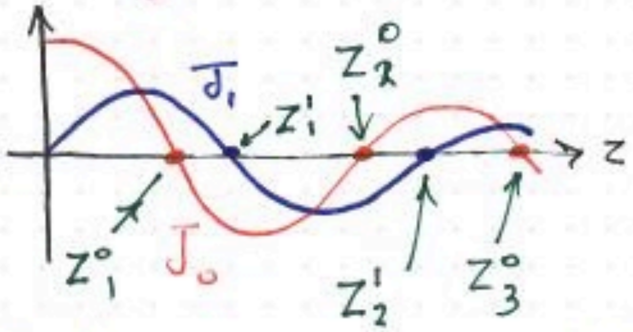
Independent sols. are "$J_m(z)$" and "$Y_m(z)$"

- examples of special functions

Web resource: Search for DLMF

Matlab funks:
Besselj(nu,z)
Bessely(nu,z)

The "order" m need not be an integer! More often we use ν (J_ν & Y_ν)



The J_m's are wiggly funks with plenty of zeros...

Let z_n^m be the n^{th} zero of $J_m(z)$

For large z it is known that
$$J_m(z) \approx \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{m\pi}{2} - \frac{\pi}{4}\right) \Rightarrow z_n^m \approx \left(n + \frac{m}{2} - \frac{1}{4}\right)\pi$$

Other props.

$J_0(0) = 1$, $J_m \propto z^m$ for $z \to 0$

Convention assigns a particular amplitude: $Y_0 \propto \ln z$, $Y_m \propto z^{-m}$ for $z \to 0$

J_0, J_m are the regular sols
Y_0, Y_m are the irreg sols.
} to Bessel's Eq.

For our drum PDE problem, $R(r) = J_m(\sqrt{\lambda}\, r)$ $m = 0$ or $1, 2, \ldots$

can add an arbitrary const.

But $u(1,\theta,t) = 0 \Rightarrow R(1) = 0$
$$\Rightarrow \lambda = \left(z_n^m\right)^2$$

More general version of ODE:
$$x^2 y'' + (1-2a)xy' + \left(\omega^2 q^2 x^{2q} + a^2 - \nu^2 q^2\right)y = 0$$

Sols: $$y = x^a J_\nu(\omega x^q) \text{ or } x^a Y_\nu(\omega x^q)$$
(times any constant)