More Laplace transforms ...

$f(x)$	$\bar{f}(s)$
t^n	$n!/s^{n+1}$
e^{at}	$1/(s-a)$
$e^{at}f(t)$	$\bar{f}(s-a)$
$f(t-a)H(t-a)$	$e^{-as}\bar{f}(s)$

shifting theorems

eg.

$$\mathcal{L}\{te^{-t}\} = \frac{1}{(s+1)^2}$$ ← (integr. by parts or by shifting theorems)

BC in PDE problem

eg.

$$\mathcal{L}^{-1}\left\{\frac{e^{-ax}}{(s+1)^2}\right\}$$ (2nd shifting thm.)

$$= f(t-a)H(t-a)$$

with $f(t) = te^{-t}$

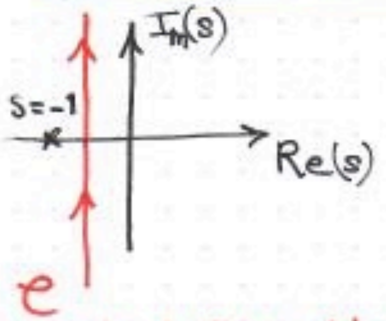
$$= (t-a)e^{t-a}H(t-a)$$ ← what we have in PDE problem with $a \leftrightarrow x$

PDE sol →

$$u(x,t) = e^{2x-t}(t-x)H(t-x)$$

By substituting into PDE, IC & BC we can demonstrate this works.

Formal Definition of Inverse Laplace Transform (behold the beast)



eg. $\frac{1}{(s+1)^2}e^{st}$ ↑ singular for $s=-1$.

$$\mathcal{L}^{-1}\{\bar{f}(s)\} = \int_{\mathcal{C}} e^{st}\bar{f}(s)\frac{ds}{2\pi i}$$
$$\equiv f(t)$$

$\mathcal{C}$ must lie to the right of $s=-1$, but the precise position is not important (Cauchy's Theorem)

$\mathcal{C}$ is a path on the complex s-plane that proceeds from $\mathrm{Im}(s) \to -\infty$ to $\mathrm{Im}(s) \to +\infty$ & lies to the right of any singularities in the integrand.

Inverse Laplace transforms can therefore be computed by suitably adding additional curves to the Bromwich contour to complete a closed loop, and then using Cauchy's Theorem (residue calculus).

Additional Notes: $\bar{f}(s)$ only makes sense for certain ranges of s

eg. $\mathcal{L}\{e^{at}\} = \left[\frac{-e^{-(s-a)t}}{(s-a)}\right]_0^\infty$ which only exists for $\mathrm{Re}(s) > a$.

(always requires that $\mathrm{Re}(s)$ is larger than something for any f)

This is why the Bromwich contour must lie to the right of the singularities.