d'Alembert's solution directly: $u_{tt} = c^2 u_{xx}$

Put $\xi = x - ct$, $\eta = x + ct$ & change variables $(x,t) \to (\xi, \eta)$

$\frac{\partial}{\partial x} \to \frac{\partial}{\partial \eta} + \frac{\partial}{\partial \xi}$, $\frac{\partial}{\partial t} = c\left(\frac{\partial}{\partial \eta} - \frac{\partial}{\partial \xi}\right)$. The wave eq. is then

$$4c^2 u_{\xi\eta} = 0$$

with gen. sol. $u = F(\xi) + G(\eta)$

Incorporating the ICs gives F & G in terms of f & g such that

$f = u(x,0)$, $g = u_t(x,0)$

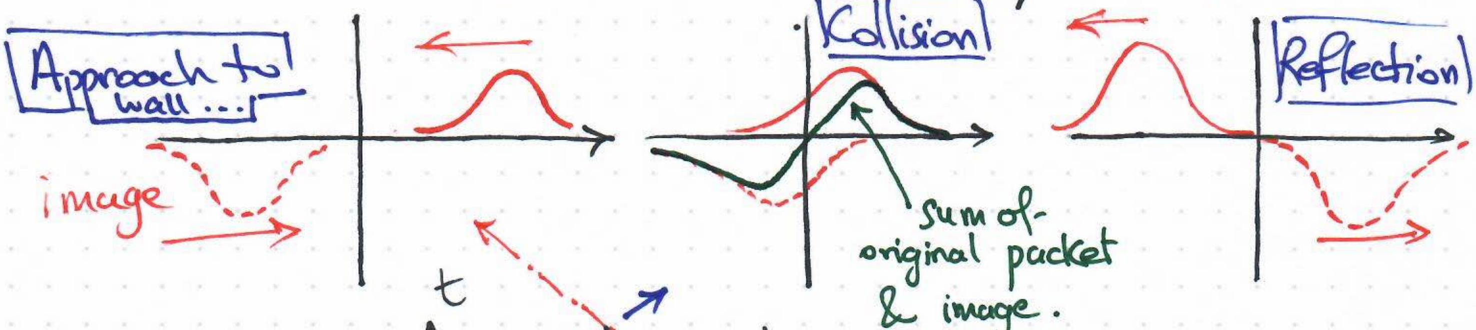
$$u(x,t) = \frac{1}{2} f(x-ct) + \frac{1}{2} f(x+ct) + \frac{1}{2c} \int_{x-ct}^{x+ct} g(z)\,dz .$$

eg. $g = 0$ (plucked string)

solution splits equally into packets travelling in opposite directions at speed c

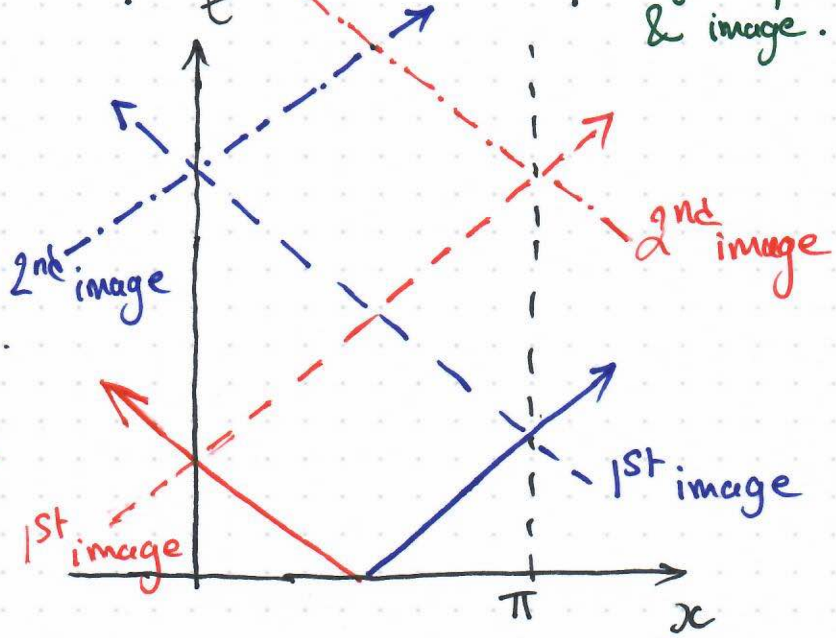
But this is the solution for an infinite line.

For $u = 0$ at $x = 0$, we must add an equal & opposite image



Including both walls & subsequent reflections...

(space-time diagram)



Full solution requires an infinite, periodic array of images

Equal & opposite image/packet pairs $\Rightarrow$ <u>odd</u> function

This is the physical interpretation of our odd, <u>periodic</u> extension.

And now I see with eye serene,
the very pulse of the machine;
A being breathing thoughtful breath,
A traveller between life & death,
The reason firm, the temperate will,
Endurance, foresight, strength & skill.

Is anyone reading these notes?
Am I wasting my time?
Should I stop?