Q1

* $\dot{N} > 0$ for $N = N_0$, so $N(t)$ must increase.
$\dot{N} > 0$ for all $N_0 \leq N < \gamma/\delta$, so $N(t)$ must continue to increase until $N = \gamma/\delta$. At that stage $\dot{N} \to 0$ and so $N(t)$ remains at $N = \gamma/\delta$.

* The ODE is 1st-order and separable. $\therefore$

$$\int \frac{dN}{dt} \frac{dt}{N(\gamma - \delta N)} = \int dt + C$$

arb. const. of integration.

Quick partial fraction: $\dfrac{1}{N(\gamma-\delta N)} = \dfrac{a}{N} + \dfrac{b}{\gamma - \delta N}$ with $a = 1/\gamma$, $b = \delta/\gamma$

Hence

$$\frac{1}{\gamma}\int \frac{dN}{N} + \frac{\delta}{\gamma}\int \frac{dN}{\gamma - \delta N} = t + C$$

But $\delta N < \delta$ (from above), so that

$$\frac{1}{\gamma} \ln N + \frac{1}{\gamma}\left(-\ln(\gamma - \delta N)\right) = t + C$$

OR $$\frac{N}{\gamma - \delta N} = E e^{\gamma t} \quad \text{with } E = e^{\gamma C}$$ (another arb. const.)

At $t = 0$, $N = N_0$, and so $E = \dfrac{N_0}{\gamma - \delta N_0}$.

But $N = \gamma E e^{\gamma t} - \delta E e^{\gamma t} N$

$$\rightarrow \quad N = \frac{\gamma E e^{\gamma t}}{1 + \delta E e^{\gamma t}} \equiv \frac{\gamma N_0 e^{\gamma t}}{\gamma - \delta N_0 + \delta N_0 e^{\gamma t}}$$

which satisfies $N = N_0$ at $t = 0$

& $N \to \gamma/\delta$ for $t \to \infty$ $(e^{\gamma t} \to \infty)$