Newton's law states (Q2)

spring force ($k \times$ extension)

$$m\ddot{x} = -kx + mg\cos\theta$$

mass $\times$ acceleration; gravity force

i.e. $\ddot{x} + \omega_0^2 x = G$ with $\omega_0^2 = k/m$, $G = g\cos\theta$

Initial conditions: $x(0) = 0$

$\dot{x}(0) = V$.

Homogeneous Sols.: pose $Ae^{mt} \Rightarrow m^2 = -\omega_0^2$ or $m = \pm i\omega_0$

Real form of solution: $x_h = C\cos\omega_0 t + D\sin\omega_0 t$

C, D arb. consts.

Particular Sol.: The RHS is a constant (polynomial of degree zero), so a constant as trial part. sol. should suffice: $x_p = d$

Then $\omega_0^2 d = G$ so $d = G/\omega_0^2$.

Gen. Sol. $x = x_h + x_p = C\cos\omega_0 t + D\sin\omega_0 t + \frac{G}{\omega_0^2}$

$x(0) = 0 \Rightarrow C = -G/\omega_0^2$

$\dot{x}(0) = V \Rightarrow \omega_0 D = V$

Hence $$x(t) = \frac{G}{\omega_0^2}\left(1 - \cos\omega_0 t\right) + \frac{V}{\omega_0}\sin\omega_0 t$$

If $x = A - R\cos(\alpha t + \gamma)$, then

$\boxed{A = \frac{G}{\omega_0^2}}$ and $-R\cos\alpha t\cos\gamma + R\sin\alpha t\sin\gamma$

$= -\frac{G}{\omega_0^2}\cos\omega_0 t + \frac{V}{\omega_0}\sin\omega_0 t$

$\longrightarrow R\cos\gamma = G/\omega_0^2$, $R\sin\gamma = V/\omega_0$, $\boxed{\alpha \equiv \omega_0}$.

$\Rightarrow R^2 = \left(\frac{G}{\omega_0^2}\right)^2 + \left(\frac{V}{\omega_0}\right)^2$ or $\boxed{R = \sqrt{\frac{G^2}{\omega_0^4} + \frac{V^2}{\omega_0^2}}}$ $\left(> \frac{G}{\omega_0^2}\right)$

and $\boxed{\tan\gamma = \frac{\omega_0 V}{G}}$.